問題解答

2.　力の釣合い

［問題 2.2］

(1) $\sum V = 0 : N_{AC}\sin\theta - 2 = 0, \therefore N_{AC} = 3.333\,\mathbf{N},\quad \sum H = 0 : -3.333\cos\theta - N_{BC} = 0, \therefore N_{BC} = -2.667\,\mathbf{N}$

(2) $\sum V = 0 : -N_{AC}\sin 30° - N_{BC}\sin 60° - 2 = 0,\quad \sum H = 0 : N_{AC}\cos 30° + N_{BC}\cos 60° = 0,$

上第 2 式より $N_{AC} = -N_{BC}\dfrac{\cos 60°}{\cos 30°}$，これと第 1 式より $N_{AC} = 2.0\text{N},\ N_{BC} = -3.464\text{N}$

(3) $\sum V = 0 : N_{AC}\sin 60° + N_{BC}\sin 30° - 3 = 0,\quad \sum H = 0 : -N_{AC}\cos 60° + N_{BC}\cos 30° = 0,$

上第 2 式より $N_{AC} = N_{BC}\dfrac{\cos 30°}{\cos 60°}$，これと第 1 式より $N_{AC} = 2.598\text{N},\ N_{BC} = 1.5\text{N}$

［問題 2.3］

(1) $\sum M_B = 0 : 5R_A - 10\cdot 2 = 0,\ \therefore R_A = 4.0\text{N},\quad \sum V = 0 : R_A - 10 + R_B = 0,\ \therefore R_B = 6.0\text{N}$

(2) $\sum M_B = 0 : -5\cdot 9 + 6R_A - 3\cdot 3 = 0,\ \therefore R_A = 9.0\text{N},$

$\sum V = 0 : -5 + R_A - 3 + R_B = 0,\ \therefore R_B = -1.0\text{N}$

(3) $\sum M_B = 0 : 5R_A - 12\sin 60°\cdot 4 - 10\cdot 2 = 0,\ \therefore R_A = 12\cdot 314\text{N}$

$\sum V = 0 : R_A - 12\sin 60° - 10 + R_B = 0,\ \therefore R_B = 8.078\text{N},$

$\sum H = 0 : H_A = -12\cos 60° = 0,\ \therefore H_A = 6.0\text{N}$

(4) $\sum M_A = 0 : 3\cdot 2 + 5 - R_B\cdot 8 = 0,\ \therefore R_B = 11/8 = 1.375\text{N}$

$\sum M_b = 0 : -R_A\cdot 8 - 3\cdot 6 + 5 = 0,\ \therefore R_A = -13/8 = -1.625\text{N}$

3.　力の合成・分解

［問題 3.1］

(1) $R = 5 - 3 + 4 = 6\text{kN}$　(右向き)

(2) $R = \sqrt{(4 + 3\cos 60°)^2 + (3\sin 60°)^2} = \sqrt{37} = 6.083\text{kN}$

$\theta = \tan^{-1}\left(\dfrac{3\sin 60°}{4 + 3\cos 60°}\right) = \tan^{-1}\left(\dfrac{3\sqrt{3}}{11}\right) = 25°17'05''$　(P_2 から左回りに $25°17'05''$ の方向)

(3) $R = \sqrt{(4 + 3\cos 120°)^2 + (3\sin 120°)^2} = \sqrt{13} = 3.606\text{kN}$

$\theta = \tan^{-1}\left(\dfrac{3\sin 120°}{4 + 3\cos 120°}\right) = \tan^{-1}\left(\dfrac{3\sqrt{3}}{5}\right) = 46°06'07''$　(P_2 から左回りに $46°06'07''$ の方向)

(4) R=6.59kN,　$\tan\alpha = 0.754$

［問題 3.2］

(4) $R = \sqrt{(4\cos 60° + 3\cos 45° - 3\cos 30°)^2 + (4\sin 60° + 3\sin 30° - 3\sin 45°)^2} = 3.225\text{kN}$

$\theta = \tan^{-1}\left(\dfrac{4\sin 60° + 3\sin 30° - 3\sin 45°}{4\cos 60° + 3\cos 45° - 3\sin 30°}\right) = 61°48'58''$　(x 軸から $61°48'58''$ の方向)

(5) $R = \sqrt{(3\cos 30° - 4\cos 45° - 3\cos 60° + 2\cos 30°)^2 + (3\sin 30° + 4\sin 45° - 3\sin 60° - 2\sin 30°)^2}$

$= 0.733\text{kN}$

$\theta = \tan^{-1}\left(\dfrac{3\sin 30° + 4\sin 45° - 3\sin 60° - 2\sin 30°}{3\cos 30° - 4\cos 45° - 3\cos 60° + 2\cos 30°}\right) = 89°51'59''$　(x 軸から $89°51'59''$ の方向)

［問題 3.3］　(省略)

［問題 3.4］

(1) $P_x = 5\cos 30° = 4.33\text{kN}$（左向き），　$P_y = 5\sin 30° = 2.50\text{kN}$（下向き）

(2) $\left.\begin{array}{l} P_x + P_y \cos 60° = 5\cos 30° \\ P_y \sin 60° = 5\sin 30° \end{array}\right\}$ これより　$P_x = 2.887\text{kN}$,　$P_y = 2.887\text{kN}$

(3) $\left.\begin{array}{l} P_x - P_y \cos 60° = 4\cos 45° \\ P_y \sin 60° = 4\sin 45° \end{array}\right\}$ これより ,　$P_x = 4.461\text{kN}$,　$P_y = 3.266\text{kN}$

4.　モーメント

［問題 4.1］

(1) $M_\text{O} = 2\cdot 3 = 6\text{Nm}$

(2) $M_\text{O} = -2\cdot 3 = -6\text{Nm}$

(3) $M_\text{O} = 3\cdot 3 - 2\cdot 4 + 4\cdot 2 = 9\text{Nm}$

(4) $M_\text{O} = 2\cdot 5 - 3\cdot 3 = 1\text{Nm}$

(5) $M_\text{O} = -8\sin 60°\cdot 3 - 8\cos 60°\cdot 4 + 5\sin 45°\cdot 0 - 5\cos 45°\cdot 3 = -47.391\text{Nm}$

［問題 4.2］

(1) $R = 3 + 5 = 8\text{kN}$　(下向き),　$x = \dfrac{5\cdot 8}{8} = 5\text{m}$　(点 O から右へ 5m の位置)

(1) $R = 3 - 4 + 3 + 3 = 5\text{kN}$ (下向き),　$x = \dfrac{-4\cdot 3 + 3\cdot 6 + 3\cdot 10}{5} = 7.2\text{m}$　(点 O から右へ 7.2m の位置)

(2) $R = 4 - 3 + 2 + 3 = 6\text{kN}$ (下向き),　$x = \dfrac{-4\cdot 8 + 3\cdot 5 - 2\cdot 2 + 3\cdot 2}{6} = -2.5\text{m}$　(点 O から左へ 2.5m の位置)

(4) $R = \sqrt{3^2 + (-4 + 3 - 2 - 3)^2} = 3\sqrt{5} = 6.708\text{kN}$

$\tan\theta = \dfrac{-4 + 3 - 2 - 3}{-3} = 2,\quad \theta = \tan^{-1}(2) = 63°26'05''\ \ or\ \ 243°26'05''$

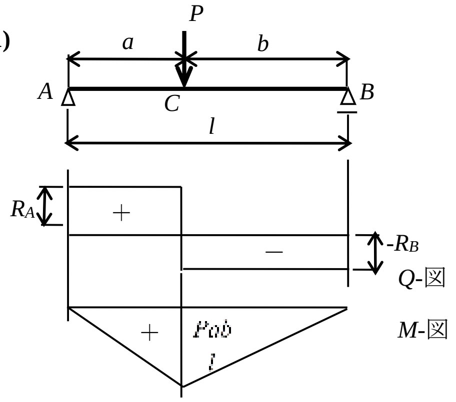
合力 R は左斜め下がりであるから，水平軸より 243°26′05″ の向き

$x = \dfrac{-3\cdot 4 - 4\cdot 8 + 3\cdot 5 - 2\cdot 2 + 3\cdot 2}{6.708} = -4.025\text{m}$,　点 O から合力までの垂直距離 4.025m

5.　静定ばり

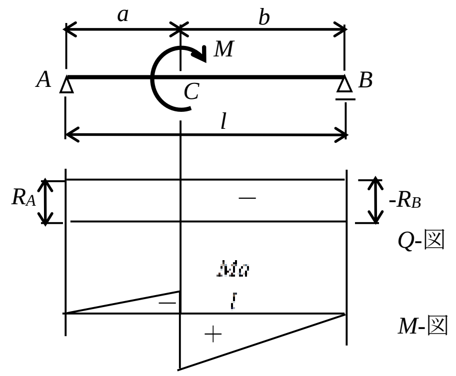
［問題 5.1］

(1)



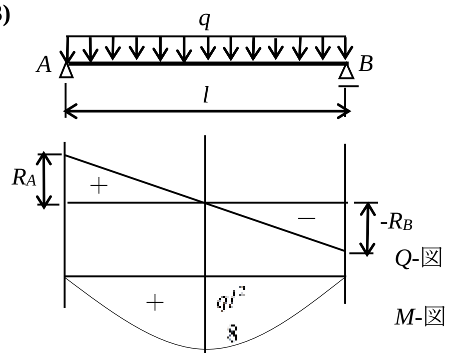
$$R_A = \frac{Pb}{l} \qquad R_B = \frac{Pa}{l}$$

(2)



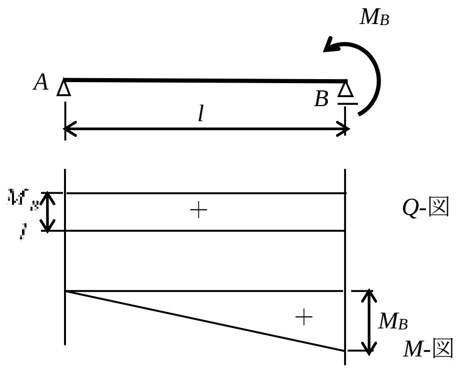
$$R_A = -\frac{M}{l} \qquad R_B = \frac{M}{l}$$

(3)



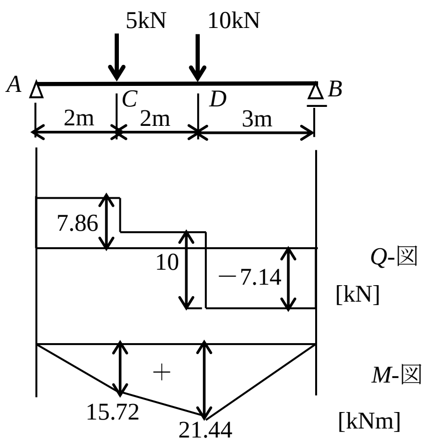
$$R_A = R_B = \frac{ql}{2}$$

(4)



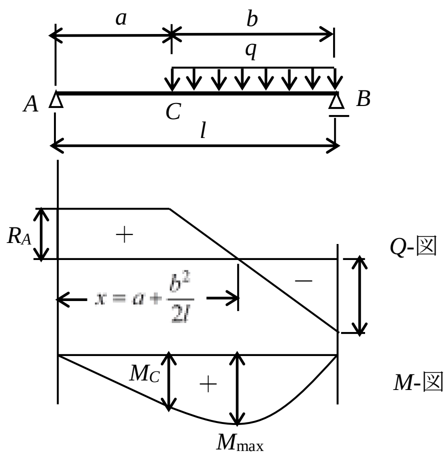
$$R_A = \frac{M_B}{l},\ R_B = -\frac{M_B}{l}$$

(5)



$$R_A = 7.86\text{kN},\ R_B = 7.14\text{kN}$$

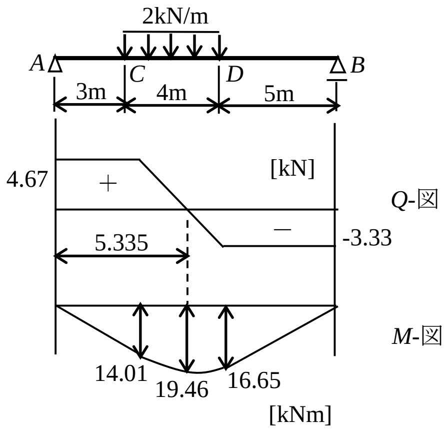
(6)



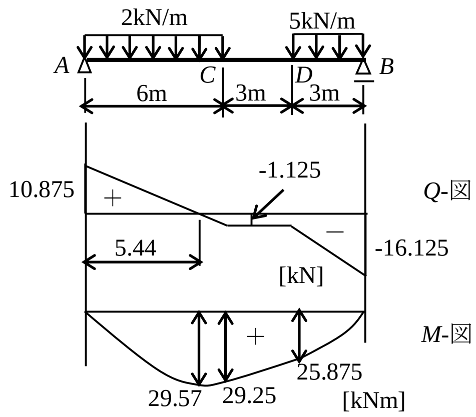
$$R_A = \frac{qb^2}{2l} \qquad R_B = \frac{qb}{l}\left(a + \frac{b}{2}\right)$$

$$M_C = \frac{qab^2}{2l} \qquad M_{\max} = \frac{qb^2}{8l^2}(l+a)^2$$

(7)



(8)



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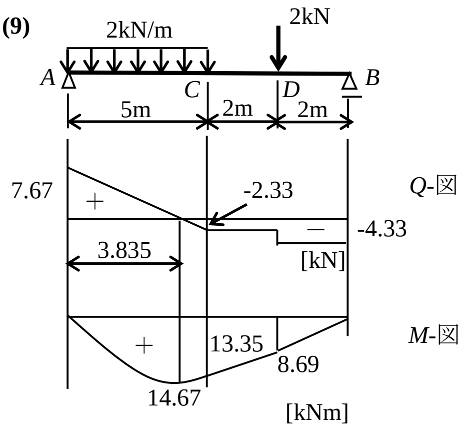
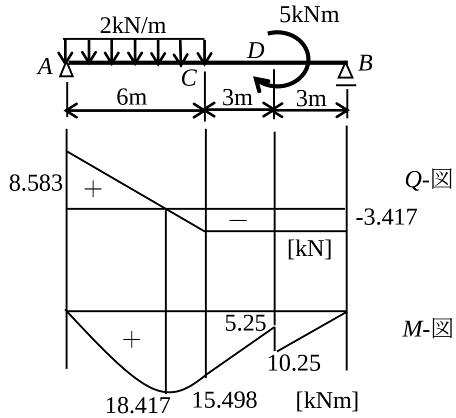


(10)



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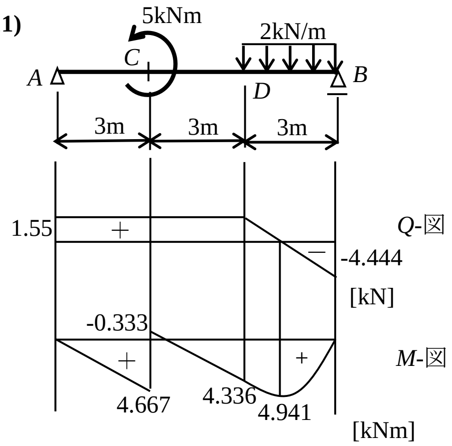

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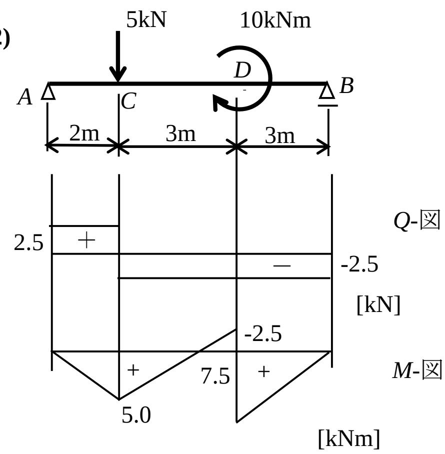

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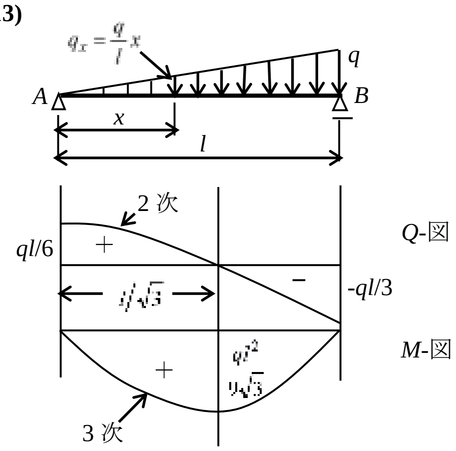
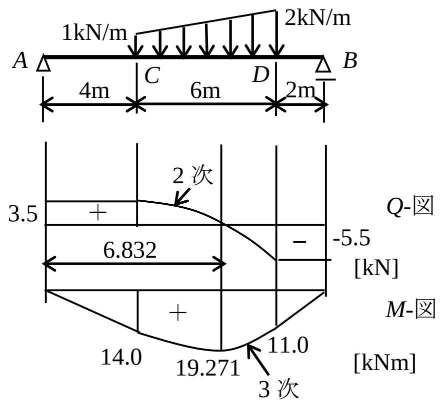
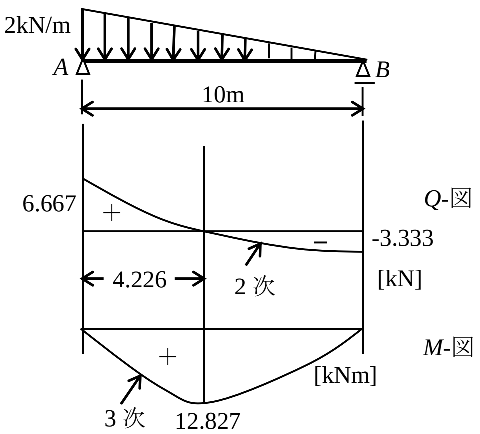


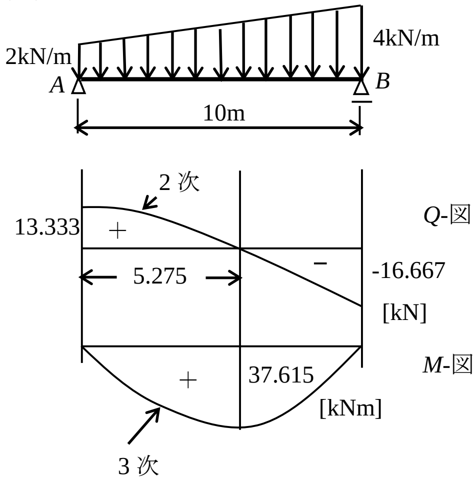
(14)



(15)

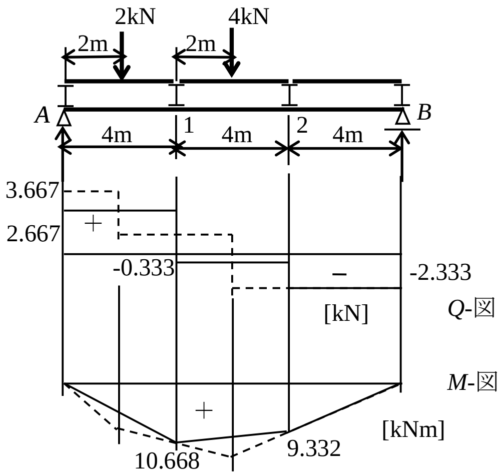


(16)

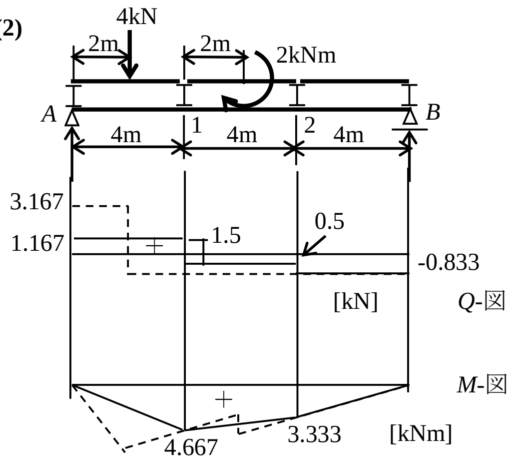


［**問題 5.2**］

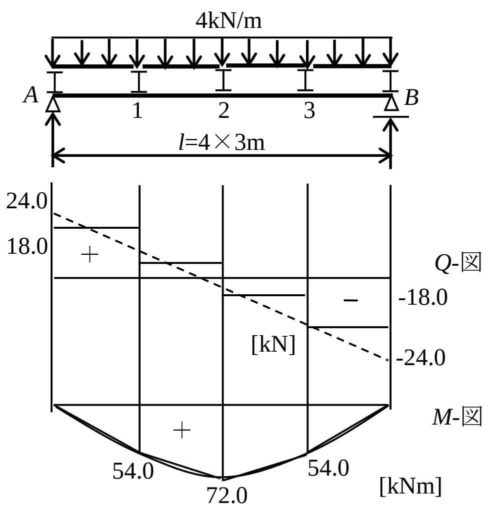
(1)



(2)

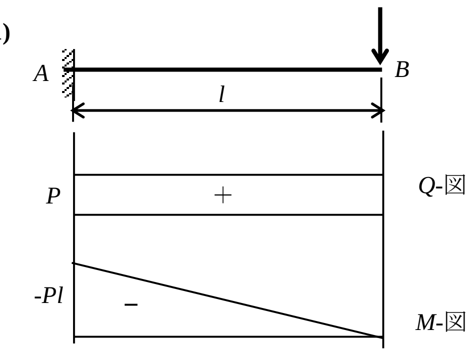


(3)

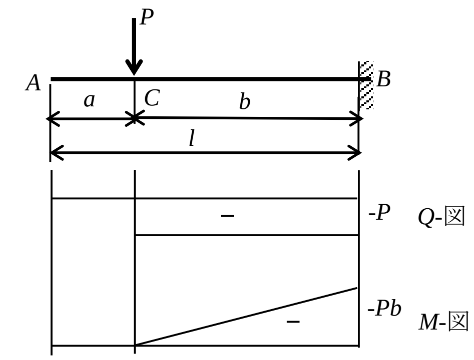


［問題 5.3］

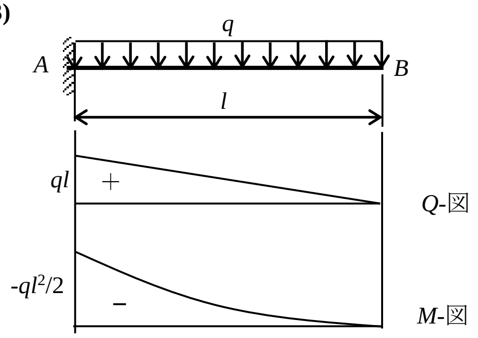
(1)



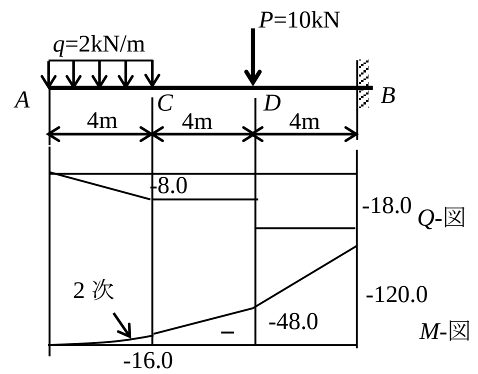
(2)



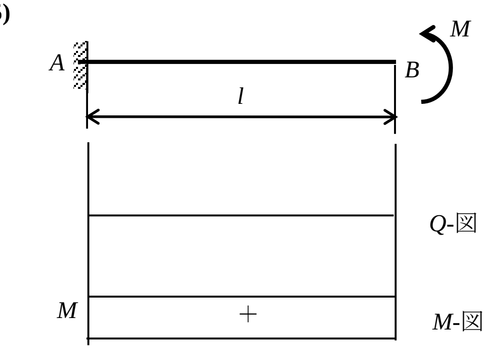
(3)



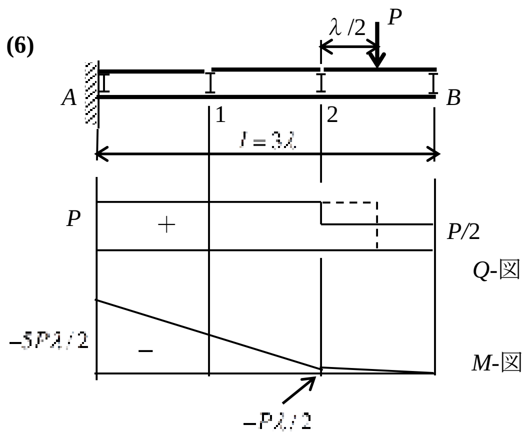
(4)



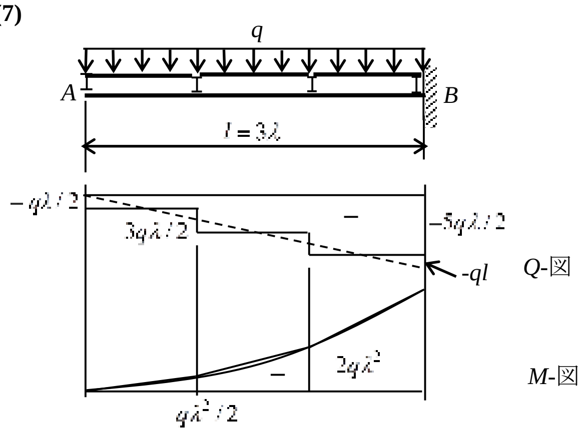
(5)



(6)

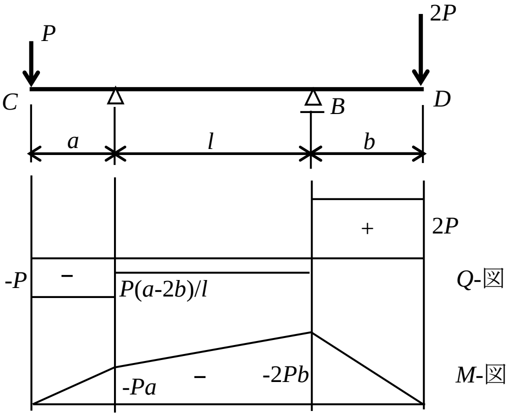


(7)



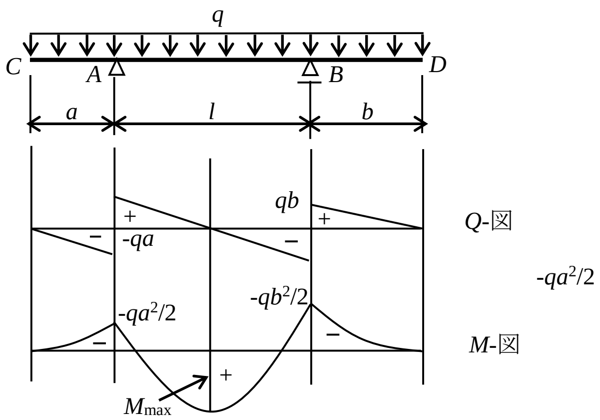
［**問題 5.4**］

(1)



$$R_A = \frac{P}{l}(a + l - 2b),\quad R_B = \frac{P}{l}(2l - a + 2b)$$

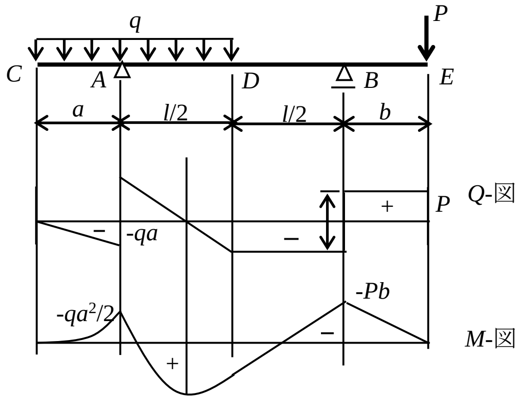
(2)



$$R_A = \frac{q}{2l}\left\{(a+l)^2 - b^2\right\},\quad R_B = \frac{q}{2l}\left\{(b+l)^2 - a^2\right\}$$

$$x = \{(a+l)^2 - b^2\}/2l\ ,\ M_{\max} = \frac{q}{8l^2}\{(a+l)^2 - b^2\}\{(a-l)^2 - b^2\}$$

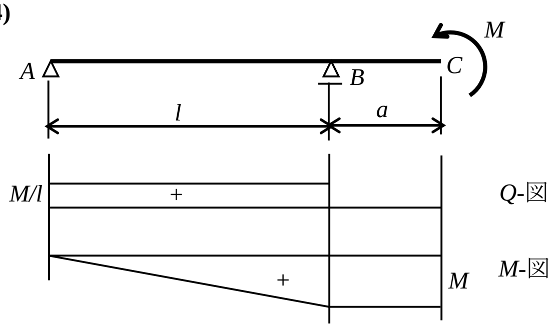
(3)



$$R_A = \frac{1}{l}\left[q(a + l/2)(a + 3l/2)/2 - Pb\right]$$

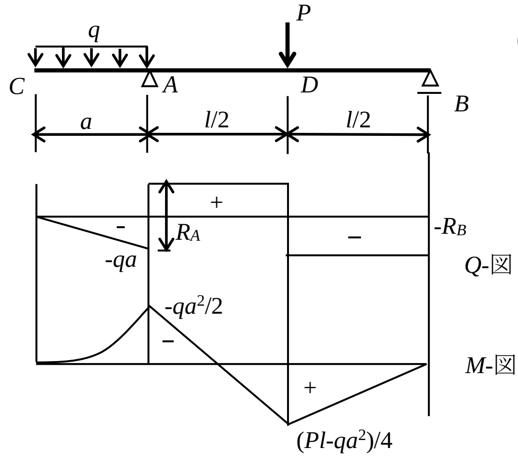
$$R_B = \frac{1}{l}\left[q(a + l/2)(l/2 - a)/2 + P(l + b)\right]$$

(4)



$$R_A = \frac{M}{l} = -R_B$$

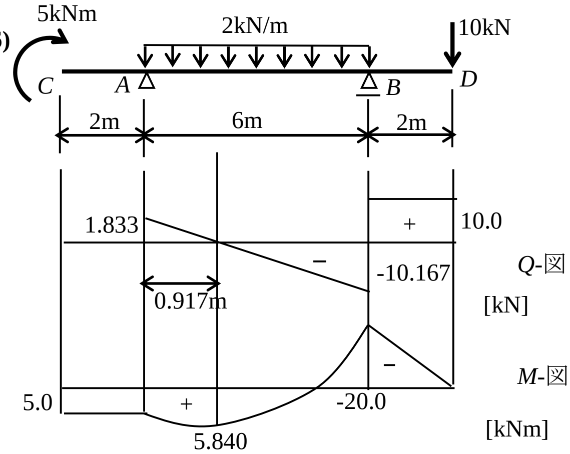
(5)



$$R_A = \frac{1}{l}\{qa(a/2 + l) + Pl/2\}$$

$$R_B = \frac{1}{l}\left\{-qa^2/2 + Pl/2\right\}$$

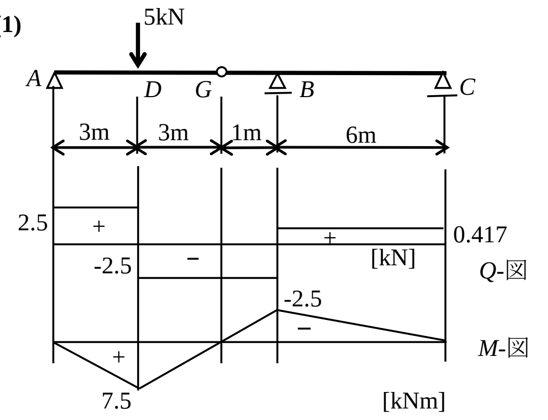
(6)



$$R_A = 1.833\text{kN},\quad R_B = 20.167\text{kN}$$

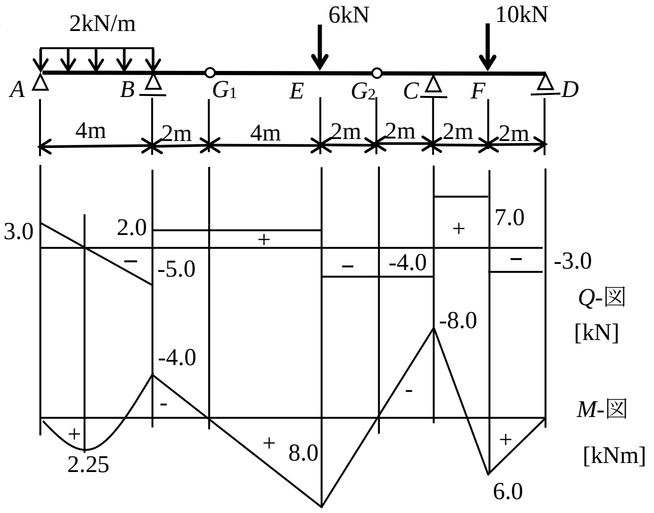
［**問題 5.5**］

(1)



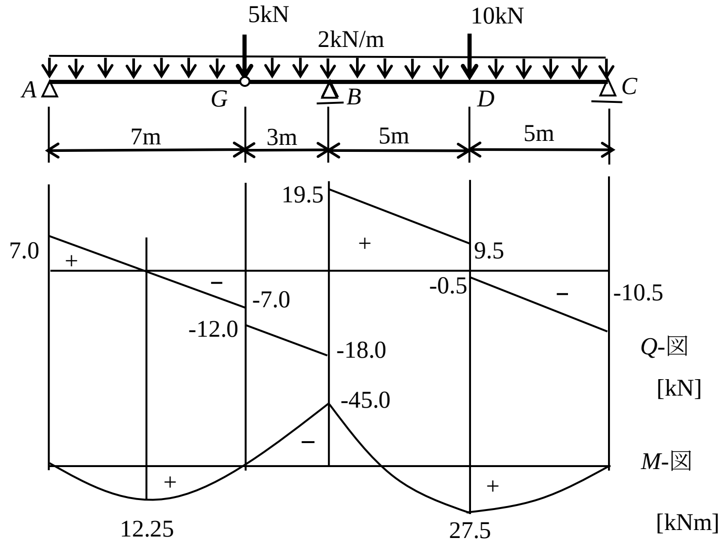
5kN
A
D
G
B
C
3m
3m
1m
6m
2.5
+
-2.5
−
+
0.417
[kN]
Q-図
-2.5
−
M-図
+
7.5
[kNm]

(2)



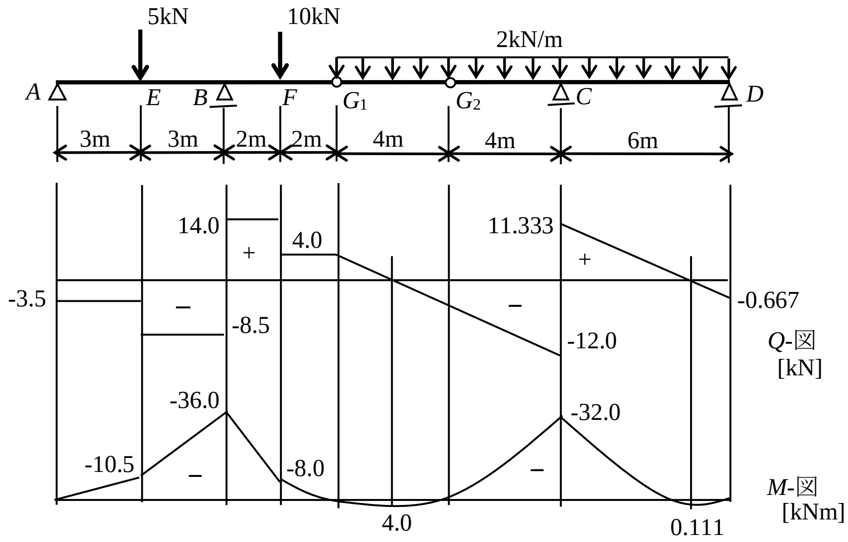
2kN/m
6kN
10kN
A
B
G1
E
G2
C
F
D
4m
2m
4m
2m
2m
2m
2m
3.0
2.0
+
−
-5.0
−
-4.0
7.0
+
−
-3.0
Q-図
[kN]
-8.0
-4.0
-
-
M-図
+
2.25
+
8.0
+
6.0
[kNm]

(3)



5kN
10kN
2kN/m
A
G
B
D
C
7m
3m
5m
5m
19.5
+
7.0
+
−
-7.0
9.5
-0.5
−
-10.5
-12.0
-18.0
Q-図
[kN]
-45.0
−
M-図
+
+
12.25
27.5
[kNm]

(3)

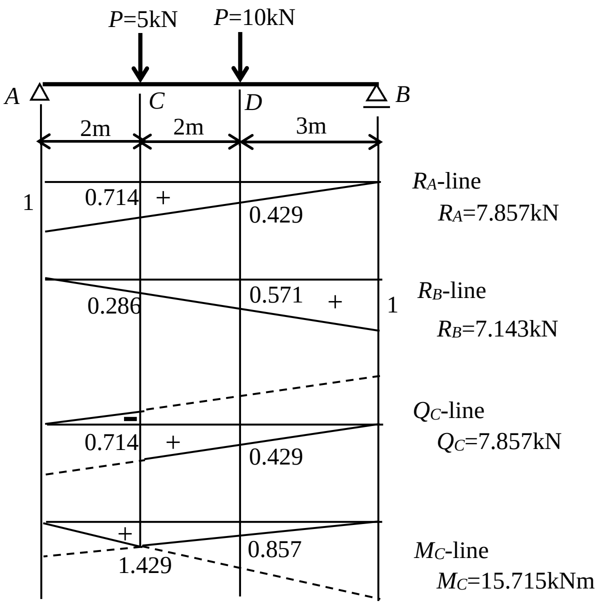


5kN
10kN
2kN/m
A
E
B
F
G1
G2
C
D
3m
3m
2m
2m
4m
4m
6m
14.0
+
4.0
11.333
+
-3.5
−
-8.5
−
-0.667
-12.0
Q-図
[kN]
-36.0
-32.0
-10.5
−
-8.0
−
M-図
[kNm]
4.0
0.111

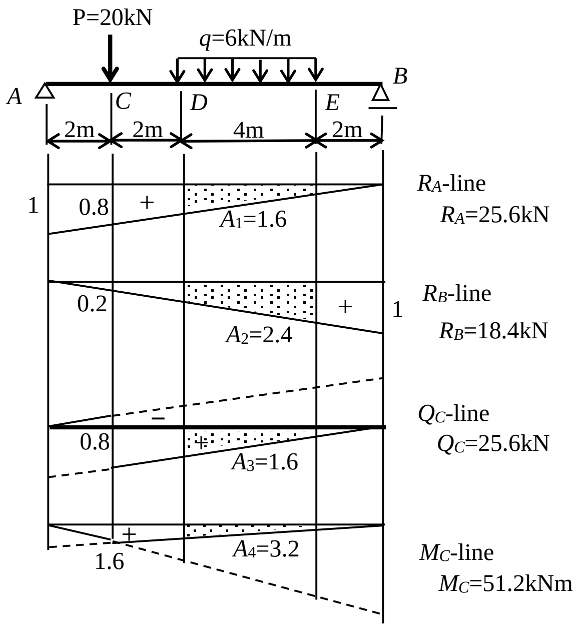
6.　はりの影響線

［問題 6.1］

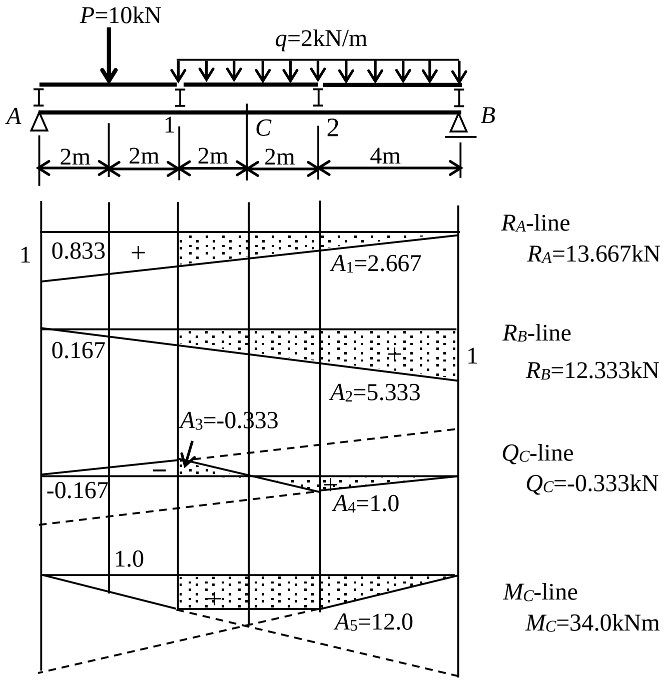
(1)



(2)

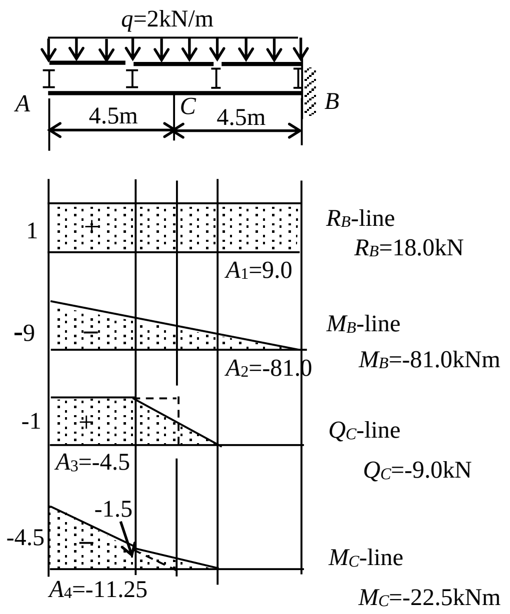


(3)



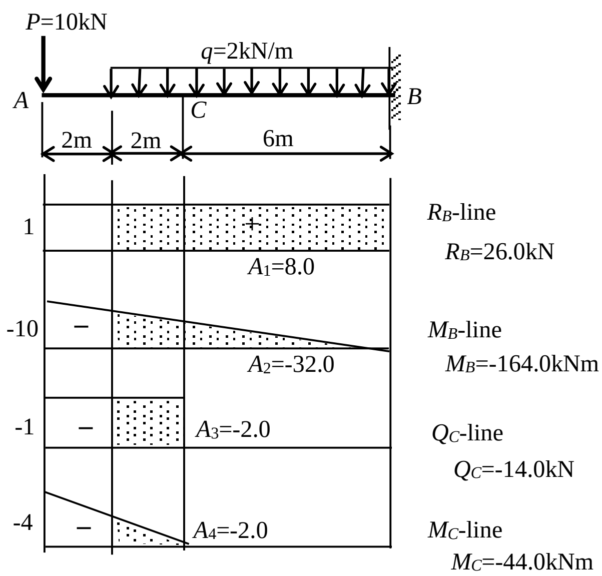
［問題 **6.2**］

(1)



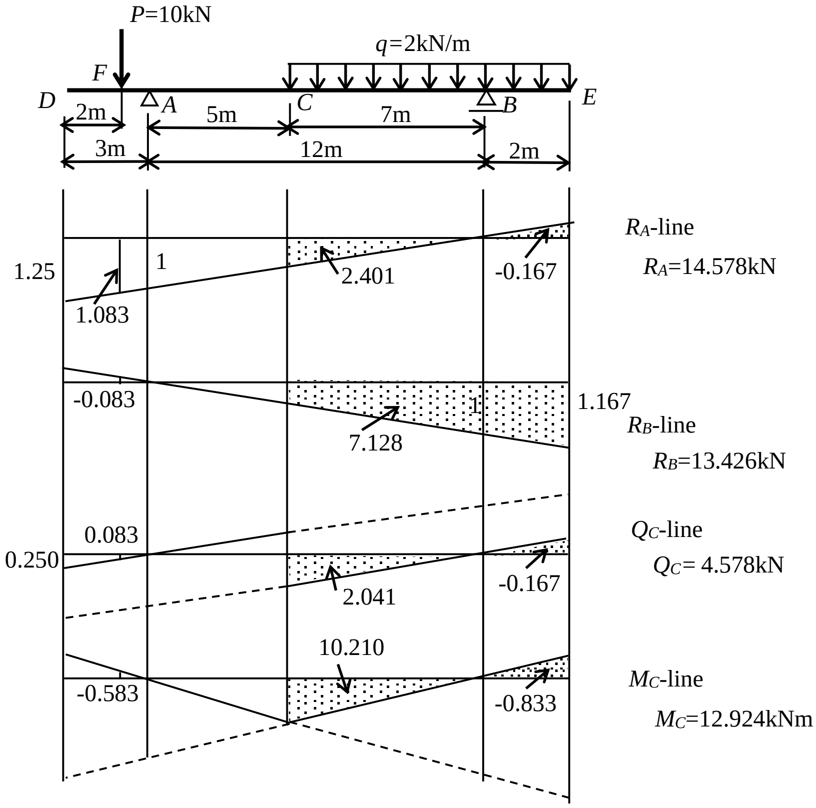
q=2kN/m
A
C
B
4.5m
4.5m
1
+
RB-line
RB=18.0kN
A1=9.0
-9
−
MB-line
MB=-81.0kNm
A2=-81.0
-1
+
QC-line
QC=-9.0kN
A3=-4.5
-1.5
-4.5
−
MC-line
A4=-11.25
MC=-22.5kNm

(2)



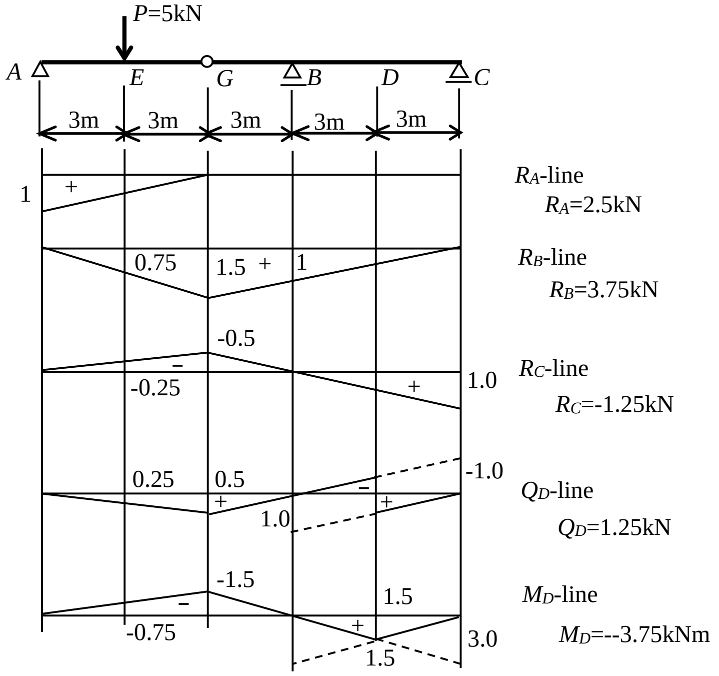
P=10kN
q=2kN/m
A
C
B
2m
2m
6m
1
+
RB-line
RB=26.0kN
A1=8.0
-10
−
MB-line
A2=-32.0
MB=-164.0kNm
-1
−
A3=-2.0
QC-line
QC=-14.0kN
-4
−
A4=-2.0
MC-line
MC=-44.0kNm

［問題 **6.3**］



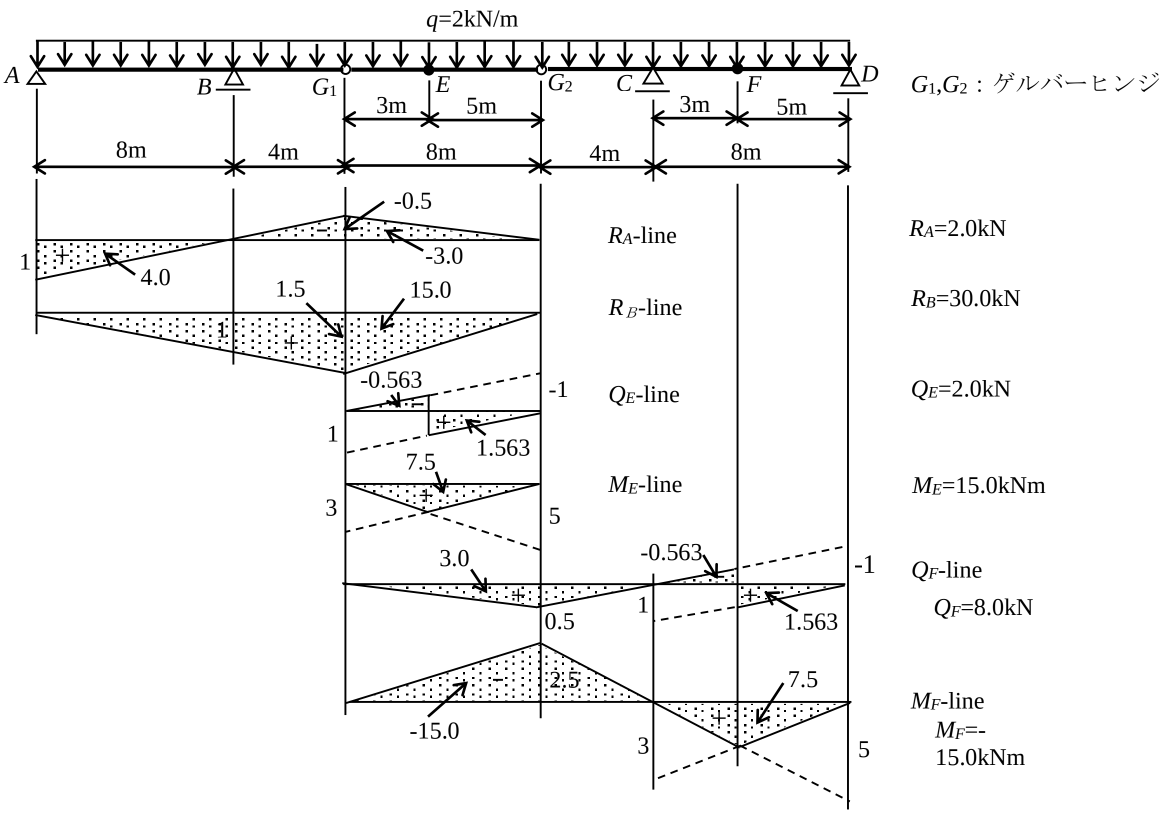
P=10kN
q=2kN/m
F
D
A
C
B
E
2m
5m
7m
3m
12m
2m
RA-line
RA=14.578kN
1.25
1
2.401
-0.167
1.083
-0.083
1.167
1
7.128
RB-line
RB=13.426kN
0.083
0.250
QC-line
QC= 4.578kN
2.041
-0.167
10.210
MC-line
-0.583
-0.833
MC=12.924kNm

［**問題 6.4**］



P=5kN
A
E
G
B
D
C
3m
3m
3m
3m
3m
RA-line
RA=2.5kN
RB-line
RB=3.75kN
RC-line
RC=-1.25kN
QD-line
QD=1.25kN
MD-line
MD=--3.75kNm
1
+
0.75
1.5
1
-0.5
−
-0.25
1.0
0.25
0.5
-1.0
-1.5
1.5
-0.75
3.0

［**問題 6.5**］



q=2kN/m
A
B
G1
E
G2
C
F
D
G1,G2：ゲルバーヒンジ
3m
5m
3m
5m
8m
4m
8m
4m
8m
RA-line
RA=2.0kN
RB-line
RB=30.0kN
QE-line
QE=2.0kN
ME-line
ME=15.0kNm
QF-line
QF=8.0kN
MF-line
MF=-15.0kNm
-0.5
-3.0
4.0
1.5
15.0
-0.563
1.563
7.5
3.0
0.5
-15.0
2.5

7. 部材断面の性質

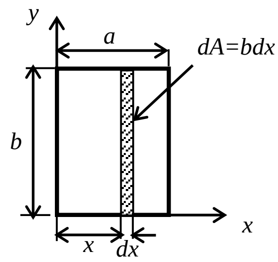


図 7.2

［問題 7.1］

断面積：$A=ab$,　　微小面積：$dA=bdx$

y 軸に対する断面 1 次モーメント $G_y=\int xdA=\int_0^a bxdx=\dfrac{a^2b}{2}$

［問題 7.2］

$A=\dfrac{1}{2}ab$,　　x 点の長さ $b'=\dfrac{b}{a}(a-x)$ であるから

$$dA=b'\cdot dx=\frac{b}{a}(a-x)dx$$

$$G_y=\int x\,dA=\int_0^a \frac{b}{a}(a-x)x\,dx=\frac{a^2b}{6}$$

$$x_0=\frac{G_y}{A}=\left(\frac{a^2b}{6}\right)\Big/\left(\frac{ab}{2}\right)=\frac{a}{3}$$

すなわち，y 軸から 3 分の 1 の点にある．

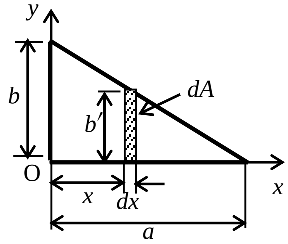


図 7.9

［問題 7.3］

(1) $y_0=\dfrac{(60\times20)\cdot50+(30\times40)\cdot20}{(60\times20)+(30\times40)}=35\text{mm}$

(2) $x_0=\dfrac{160\cdot10+160\cdot40}{160+160}=25\text{mm}$,　$y_0=\dfrac{160\cdot60+160\cdot10}{160+160}=35\text{mm}$

(3) $x_0=\dfrac{4\cdot2+5\cdot1.5+2.5\cdot2.333}{4+5+2.5}=1.855\text{m}$,　$y_0=\dfrac{4\cdot0.5+5\cdot3.5+2.5\cdot2.667}{4+5+2.5}=2.275\text{m}$

［問題 7.4］

(1) $I_{x_1}=\dfrac{bh^3}{12}$,　$I_{x_2}=\dfrac{bh^3}{12}+\dfrac{h^2}{4}\times bh=\dfrac{bh^3}{3}$,　$I_{x_3}=\dfrac{bh^3}{12}+h^2\times bh=\dfrac{13bh^3}{12}$

(2) すべて $I_x=1834.17\times10^4\ (\text{mm}^4)$

(a) $I_x=\dfrac{140\cdot130^3}{12}-2\times\dfrac{60\cdot90^3}{12}=1834.17\times10^4\text{cm}^4$　(b) $I_x=\dfrac{140\cdot130^3}{12}-\dfrac{120\cdot90^3}{12}=1834.17\times10^4\text{cm}^4$

(c) $I_x=\dfrac{140\cdot130^3}{12}-\dfrac{120\cdot90^3}{12}=1834.17\times10^4\text{mm}^4$

(3) (a)　$A=60\times20+30\times40=2\,400\text{cm}^2$,　$y_0=\dfrac{1\,200\times50+1\,200\times20}{1\,200+1\,200}=35\text{mm}$,

$$I_x=\frac{60\times20^3}{12}+(50-35)^2\times1\,200+\frac{30\times40^3}{12}+(35-20)^2\times1\,200=740\,000\text{mm}^4$$

(b)　$A=2(40\times100-30\times80)+(10\times100)=4\,200\text{mm}^2$

$G_1=50\times(100\times40)-50\times(30\times80)=80\,000\text{mm}^3$,　$G_2=105\times(10\times100)=105\,000\text{mm}^3$,

$\therefore G_x=2\times G_1+G_2=265\,000\text{mm}^3$

$$y_0=\frac{G_x}{A}=\frac{265\,000}{4\,200}=63.1\text{mm}$$

$$I_1=2\left\{\left(\frac{40\times100^3}{12}-\frac{30\times80^3}{12}\right)+13.1^2\times1600\right\}=465.58\times10^4\text{mm}^4,$$

$$I_2=\frac{100\times10^3}{12}+41.9^2\times(10\times100)=176.39\times10^4\text{mm}^4$$

$$I_{x_1}=I_1+I_2=641.97\times10^4\text{mm}^4$$

(4)

$A_1=3\times15=45\text{cm}^2$,　$A_2=12\times4=48\text{cm}^2$,　$A_3=5\times15=75\text{cm}^2$,　$\therefore\ A=168\text{cm}^2$

$G_1 = 7.5\times A_1 = 337.5\text{cm}^3$,　$G_2 = 2\times A_2 = 96\text{cm}^3$,　$\therefore G_3 = 7.5\times A_3 = 562.5\text{cm}^3$,　$G_x = 996\text{cm}^3$

$$y_0 = \frac{G_x}{A} = \frac{996}{168} = 5.93\text{cm}$$

$$I_1 = \frac{3\times 15^3}{12} + 7.5^2\times 45 = 3375\text{cm}^4,\quad I_2 = \frac{12\times 4^3}{12} + 2^2\times 48 = 256\text{cm}^4,$$

$$I_3 = \frac{5\times 15^3}{12} + 7.5^2\times 75 = 5625\text{cm}^4,\quad \therefore\ I_x = 9256\text{cm}^4,\quad I_G = 9256 - 5.93^2\times 168 = 3348.30cm^4$$

［問題 7.5］ **(1)** $I_{xy} = x_0 y_0 A = 0\cdot\frac{b}{2}\cdot A = 0$,　$\left(\text{あるいは}\quad I_{xy} = \int xydA = \int_{-a/2}^{a/2} xdx\int_0^b ydy = 0\right)$

(2) $I_{xy} = x_0 y_0 A = -\frac{a}{2}\cdot\frac{b}{2}\cdot ab = -\frac{a^2b^2}{4}$,

$$\left(\text{あるいは}I_{xy} = \int xydA = \int_0^b ydy\int_{-a}^0 xdx = \frac{b^2}{2}\left[\frac{x^2}{2}\right]_{-a}^0 = -\frac{a^2b^2}{4}\right)$$

［問題 7.6］

$$I_u = \int v^2 dA = \int (y\cos\theta - x\sin\theta)^2 dA$$

$$= \int y^2\cos^2\theta\, dA - \int xy\cdot 2\cos\theta\sin\theta\, dA + \int x^2\sin^2\theta\, dA$$

$$= I_x\cos^2\theta + I_y\sin^2\theta - I_{xy}\sin 2\theta = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_v = \int u^2 dA = \int (x\cos\theta + y\sin\theta)^2 dA$$

$$= \int x^2\cos^2\theta\, dA + \int xy\cdot 2\sin\theta\cos\theta\, dA + \int y^2\sin^2\theta\, dA$$

$$= I_y\cos^2\theta + I_x\sin^2\theta + I_{xy}\sin 2\theta = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$I_{uv} = \int uvdA = \int (x\cos\theta + y\sin\theta)(y\cos\theta - x\sin\theta)dA$$

$$= \int xy\cos^2\theta\, dA - \int x^2\cos\theta\sin\theta\, dA + \int y^2\sin\theta\cos\theta\, dA - \int xy\sin^2\theta\, dA$$

$$= \frac{1}{2}I_x\sin 2\theta - \frac{1}{2}I_y\sin 2\theta + \int xy(\cos^2\theta - \sin^2\theta)dA = \frac{1}{2}I_x\sin 2\theta - \frac{1}{2}I_y\sin 2\theta + I_{xy}\cos 2\theta$$

$$= \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$

注）以上よりさらに次の関係が得られる．

$$\frac{\partial I_u}{\partial\theta} = 2I_x\cos\theta(-\sin\theta) + 2I_y\sin\theta\cos\theta - 2I_{xy}\cos 2\theta = -I_x\sin 2\theta + I_y\sin 2\theta - 2I_{xy}\cos 2\theta$$

$$\frac{\partial I_v}{\partial\theta} = 2I_x\sin\theta\cos\theta + 2I_y\cos\theta(-\sin\theta) + 2I_{xy}\cos 2\theta = I_x\sin 2\theta - I_y\sin 2\theta + 2I_{xy}\cos 2\theta$$

$$\therefore\ -\frac{\partial I_u}{\partial\theta} = \frac{\partial I_v}{\partial\theta} = 2I_{uv}$$

［問題 7.7］式(7.22)を 3 元 1 次の連立方程式として解く．第 1 式と第 2 式を加え合わせると

$$I_x + I_y = I_u + I_v \tag{1}$$

第 3 式より　$I_{xy} = \frac{I_{uv}}{\cos 2\theta} - \frac{I_x - I_y}{2}\tan 2\theta$　(2)

式(7.22)の第 1 式より第 2 式を引くと

$$I_u - I_v = (I_x - I_y)\cos 2\theta - 2I_{xy}\sin 2\theta = (I_x - I_y)\cos 2\theta - 2I_{uv}\tan 2\theta + (I_x - I_y)\frac{\sin^2 2\theta}{\cos 2\theta}$$

$$= \frac{I_x - I_y}{\cos 2\theta}(\cos^2 2\theta + \sin^2 2\theta) - 2I_{uv}\tan 2\theta = \frac{I_x - I_y}{\cos 2\theta} - 2I_{uv}\tan 2\theta$$

ゆえに　(3)

$$I_x - I_y = (I_u - I_v)\cos 2\theta + 2I_{uv}\sin 2\theta$$

式(1)と式(3)を加え合わせると，式(7.23)の第 1 式が，式(1)から式(3)を引くと第 2 式が得られる．
　式(2)に式(3)を代入して変形する．

$$I_{xy}=\frac{I_{uv}}{\cos 2\theta}-\frac{I_x-I_y}{2}\tan 2\theta=\frac{I_{uv}}{\cos 2\theta}-\frac{1}{2}[(I_u-I_v)\cos 2\theta+2I_{uv}\sin 2\theta]\tan 2\theta$$

$$=\frac{I_{uv}}{\cos 2\theta}-\frac{I_u-I_v}{2}\sin 2\theta-I_{uv}\frac{\sin^2 2\theta}{\cos 2\theta}=\frac{I_{uv}}{\cos 2\theta}-\frac{I_u-I_v}{2}\sin 2\theta-\frac{I_{uv}}{\cos 2\theta}+I_{uv}\cos 2\theta$$

$$=-\frac{I_u-I_v}{2}\sin 2\theta+I_{uv}\cos 2\theta$$

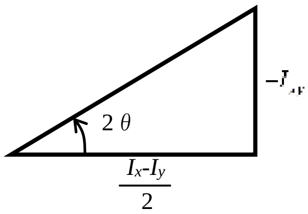
これは，式(7.23)の第 3 式である．

　［**別解**］式(7.22)の x,y と u,v を入れ替え，θ のかわりに $-\theta$ とおけばよい．

［**問題 7.8**］式(7.22)の第 3 式において I_{uv}=0 とおくと，式(7.24)

$$\tan 2\theta=-\frac{I_{xy}}{\dfrac{I_x-I_y}{2}}$$

が得られる．この式は，右図の関係を表している．したがって



$$\sin 2\theta=\frac{-I_{xy}}{\sqrt{\left(\dfrac{I_x-I_y}{2}\right)^2+I_{xy}^2}},\qquad \cos 2\theta=\frac{\dfrac{I_x-I_y}{2}}{\sqrt{\left(\dfrac{I_x-I_y}{2}\right)^2+I_{xy}^2}}$$

この関係を式(3.22)に適用すればよい．I_{uv}=0 となる I_u を I_1，I_v を I_2 とすると

$$I_1=\frac{I_x+I_y}{2}+\frac{I_x-I_y}{2}\cos 2\theta-I_{xy}\sin 2\theta=\frac{I_x+I_y}{2}+\frac{I_x-I_y}{2}\frac{\dfrac{I_x-I_y}{2}}{\sqrt{\ }}-I_{xy}\frac{-I_{xy}}{\sqrt{\ }}$$

$$=\frac{I_x+I_y}{2}+\frac{\left(\dfrac{I_x-I_y}{2}\right)^2+I_{xy}^2}{\sqrt{\ }}=\frac{I_x+I_y}{2}+\sqrt{\left(\frac{I_x-I_y}{2}\right)^2+I_{xy}^2}$$

$$I_1=\frac{I_x+I_y}{2}-\frac{I_x-I_y}{2}\cos 2\theta+I_{xy}\sin 2\theta=\frac{I_x+I_y}{2}-\frac{I_x-I_y}{2}\frac{\dfrac{I_x-I_y}{2}}{\sqrt{\ }}+I_{xy}\frac{-I_{xy}}{\sqrt{\ }}$$

$$=\frac{I_x+I_y}{2}-\frac{\left(\dfrac{I_x-I_y}{2}\right)^2+I_{xy}^2}{\sqrt{\ }}=\frac{I_x+I_y}{2}-\sqrt{\left(\frac{I_x-I_y}{2}\right)^2+I_{xy}^2}$$

［**問題 7.9**］

(1) $x_0=\dfrac{16\times1+16\times4}{16+16}=2.5\text{(cm)},\qquad y_0=\dfrac{16\times6+16\times1}{16+16}=3.5\text{(cm)}$

(2) $I_x=\dfrac{2\times8^3}{12}+6^2\times16+\dfrac{8\times2^3}{12}+1^2\times16=682.67\text{(cm}^4\text{)}$

$I_y=\dfrac{8\times2^3}{12}+1^2\times16+\dfrac{2\times8^3}{12}+4^2\times16=362.67\text{(cm}^4\text{)}$

$I_{xy}=1\times6\times16+4\times1\times16=160\text{(cm}^4\text{)}$

(3) $I_u=682.67-3.5^2\times(16+16)=290.67\text{(cm}^4\text{)}$

$I_v=362.67-2.5^2\times(16+16)=162.67\text{(cm}^4\text{)}$

$I_{uv}=160-3.5\times2.5\times(16+16)=-120\text{(cm}^4\text{)}$

(4)　$\tan 2\theta = -\dfrac{I_{uv}}{\dfrac{I_u - I_v}{2}} = -\dfrac{-120}{64} = 1.875, \quad \therefore\ \theta = 30°57'49''$

$I_{1,2} = \dfrac{I_u + I_v}{2} \pm \sqrt{\left(\dfrac{I_u - I_v}{2}\right)^2 + I_{uv}^2}$　より　$I_1 = 362.67(\mathrm{cm}^4),\quad I_2 = 90.67(\mathrm{cm}^4)$

［**問題 7.10**］

(1)　P.7-7 注意事項より求めよ．

(a)　$I = \dfrac{bh^3}{12},\ W = \dfrac{I}{h/2} = \dfrac{bh^2}{6},\ r_x = \sqrt{\dfrac{I}{A}} = \dfrac{h}{2\sqrt{3}},\ K = \dfrac{W}{A} = \dfrac{h}{6}$

(b)　$I = \dfrac{bh^3}{36},\ W_c = \dfrac{I}{2h/3} = \dfrac{bh^2}{24},\ W_t = \dfrac{I}{h/3} = \dfrac{bh^2}{12},\ r_x = \sqrt{\dfrac{I}{A}} = \dfrac{h}{3\sqrt{2}},\ K_t = \dfrac{W_c}{A} = \dfrac{h}{12},\ K_c = \dfrac{W_t}{A} = \dfrac{h}{6}$

(c)　$I = \dfrac{\pi r^4}{4},\ W = \dfrac{I}{r} = \dfrac{\pi r^3}{4},\ r_x = \sqrt{\dfrac{I}{A}} = \dfrac{r}{2},\ K = \dfrac{W}{A} = \dfrac{r}{4}$

(2)　問題 7.4(3)a を用いる．

$W_c = \dfrac{74}{2.5} = 29.6\mathrm{cm}^3,\quad W_t = \dfrac{74}{3.5} = 21.143\mathrm{cm}^3,\quad r = \sqrt{\dfrac{74}{24}} = 1.756\mathrm{cm},$

$k_c = \dfrac{21.143}{24} = 0.881\mathrm{cm},\quad k_t = \dfrac{29.6}{24} = 1.233\mathrm{cm}$

(3)　(a)　問題 7.9 を使用する．

$y_0 = 3.5cm,\quad I_x = 290.67\mathrm{cm}^4,\quad W_1 = \dfrac{290.67}{6.5} = 44.72\mathrm{cm}^3,\ W_2 = \dfrac{290.67}{3.5} = 83.05\mathrm{cm}^3$

(b)　$A = 5\times 1 + \dfrac{1}{2}\times 1\times 5 + 4 = 11.5\mathrm{m}^2,\quad y_0 = \dfrac{4\times 0.5 + 5\times 3.5 + 2.5\times 2.667}{4+5+2.5} = 2.275\mathrm{m}$

$$I_x = \frac{1\times 5^3}{12} + (3.5-2.275)^2\times(1\times 5) + \frac{1\times 5^3}{36} + \left(\frac{5}{3}+1-2.275\right)^2\times\left(\frac{1}{2}\times 1\times 5\right)$$
$$+\frac{4\times 1^3}{12} + (2.275-0.5)^2\times(4\times 1) = 10.42 + 7.5 + 3.47 + 0.38 + 0.33 + 12.60 = 34.70\mathrm{m}^4$$

$W_c = \dfrac{34.70}{3.725} = 9.32\mathrm{m}^3,\quad W_t = \dfrac{34.70}{2.275} = 15.25\mathrm{m}^3$

(4)　$A = 14\times 13 - 12\times 9 = 74\mathrm{cm}^2$

$I_x = \dfrac{14\times 13^3}{12} - \dfrac{12\times 9^3}{12} = 1834.17cm^4$

$r_x = \sqrt{\dfrac{I_x}{A}} = \sqrt{\dfrac{1834.17}{74}} = \sqrt{24.78} = 4.98\mathrm{cm}$

$x_0 = \dfrac{13\times 14\times 7 - 12\times 9\times 8}{74} = \dfrac{1274-864}{74} = 5.54\mathrm{cm}$

$$I_y = 2\left\{\frac{2\times 14^3}{12} + (7-5.54)^2\times 14\times 2\right\} + \frac{9\times 2^3}{12} + (5.54-1)^2\times 9\times 2$$
$$= 2(457.33 + 59.68) + 6 + 371.01 = 1411.03\mathrm{cm}^4$$

$r_y = \sqrt{\dfrac{I_y}{A}} = \sqrt{\dfrac{1411.03}{74}} = \sqrt{19.07} = 4.37\mathrm{cm}$

8.　応力とひずみ

［問題 8.2］　$\Delta l = \dfrac{Pl}{EA} = \dfrac{5 \times 1\,000}{2.0 \times 10^5 \times \dfrac{3.14 \times 10^2}{4}} = 3.18 \times 10^{-4}\,\mathrm{mm}$

［問題 8.3］　$\sigma = \dfrac{P}{A} = \dfrac{20\,000}{\pi \times 20^2 / 4} = 63.69\,\mathrm{N/mm^2}$，$E = \dfrac{\sigma\, l}{\Delta l} = \dfrac{63.69 \times 2\,000}{0.63} = 2.02 \times 10^5\,\mathrm{N/mm^2}$

［問題 8.4］　$\sigma = 2.0 \times 10^5 \times 0.000\,001 \times (30 - 10) = 4\,\mathrm{N/mm^2}$

［問題 8.5］　$A = \dfrac{3.14 \times 22}{4} = 379.94\mathrm{mm^2}$，　$\tau = \dfrac{P}{A} = \dfrac{1\,500}{379.94} = 3.95\,\mathrm{N/mm^2}$

［問題 8.6］3 つの部材は同じ大きさのひずみを生じるから

$$\varepsilon = \frac{P_1}{E_1 A_1} = \frac{P_2}{E_2 A_2} = \frac{P_3}{E_3 A_3}$$

一方，力の釣合いから　$P = P_1 + P_2 + P_3$

であるから　$\varepsilon = \dfrac{P_1}{E_1 A_1} = \dfrac{P_2}{E_2 A_2} = \dfrac{P_3}{E_3 A_3} = \dfrac{P_1 + P_2 + P_3}{E_1 A_1 + E_2 A_2 + E_3 A_3} = \dfrac{P}{E_1 A_1 + E_2 A_2 + E_3 A_3}$

したがって　$P_1 = \dfrac{PE_1 A_1}{E_1 A_1 + E_2 A_2 + E_3 A_3}$，$P_2 = \dfrac{PE_2 A_2}{E_1 A_1 + E_2 A_2 + E_3 A_3}$，$P_3 = \dfrac{PE_3 A_3}{E_1 A_1 + E_2 A_2 + E_3 A_3}$

すなわち，各部材の力はその部材の *EA* に比例した力が働いている．

［問題 8.7］コンクリートの断面積：$A_C = 250 \times 250 = 62\,500\mathrm{mm^2}$,

鉄筋の断面積　：$A_S = 3.14 \times 22^2 / 4 \times 4 = 1\,519.76\mathrm{mm^2}$

ここで $n = \dfrac{E_S}{E_C} = 15$ であるから

$$\sigma_C = \frac{PE_C}{E_C A_C + E_S A_S} = \frac{P}{A_C + nA_S} = \frac{50\,000}{62\,500 + 15 \times 1\,519.76} = 0.586\,\mathrm{N/mm^2}$$

$$\sigma_S = n\sigma_C = 15 \times 0.586 = 8.79\,\mathrm{N/mm^2}$$

［問題 8.8］2 つの条件がある．1 つは力の釣合いから　$\sum V = 0:\ R_1 + R_2 = P$　(1)

つぎに，部材 *AC* の伸びは部材 *CB* の縮みに等しい．すなわち　$\Delta l_1 = \Delta l_2$　(2)

ここで　$\Delta l_1 = \dfrac{R_1 l_1}{E_1 A_1}$，　$\Delta l_2 = \dfrac{R_2 l_2}{E_2 A_2}$　(3)

式(3)を式(2)に代入して　$\dfrac{R_1 l_1}{E_1 A_1} = \dfrac{R_2 l_2}{E_2 A_2}$　(4)

式(1)と式(4)より　$R_1 = \dfrac{E_1 A_1 l_2}{E_1 A_1 l_2 + E_2 A_2 l_1} P$，　$R_2 = \dfrac{E_2 A_2 l_1}{E_1 A_1 l_2 + E_2 A_2 l_1} P$　(5)

［問題 8.9］下底より *x* の点の微小要素 *dx* について考える．*x* 点に働く

応力 σ_x はその重さ $Ax \cdot \omega$ によって

$$\sigma_x = \frac{Ax \cdot \omega}{A} = x\omega$$

応力 σ_x による微小要素 *dx* の伸び量 Δdx は，Hooke の法則 $\varepsilon = \dfrac{\Delta dx}{dx}$ より

$$\Delta dx = \frac{\sigma_x}{E} dx = \frac{x\omega}{E} dx$$

したがって，全体の伸び量は，これを *x*=0 から *l* まで積分すればよい．

$$\Delta l = \int_0^l \Delta dx = \frac{\omega}{E} \int_0^l x dx = \frac{\omega}{E}\left[\frac{x^2}{2}\right]_0^l = \frac{\omega l^2}{2E} = \frac{l}{EA}\frac{W}{2}$$

ここで，$W = Al\omega$ は全自重を表す．すなわち，自重を無視して下端に W/2 の引張り力を作用させたときの伸びと等しい．

［問題 **8.10**］

部材 1 と部材 2 には共に P=2kN の力が作用する．ゆえに

$$\Delta l=\sum\frac{Pl}{EA}=\frac{P}{E}\left(\frac{l_1}{A_1}+\frac{l_2}{A_2}\right)=\frac{2\times10^3}{2.0\times10^5}\left(\frac{2\,000}{100\times100}+\frac{1\,000}{50\times50}\right)=0.006\,\text{mm}$$

9. はりの応力度

［問題 **9.1**］ **(1)** $\sigma=\dfrac{M}{I}y=\dfrac{1\,000}{\dfrac{bh^3}{12}}\times\dfrac{h}{2}=18.75\times10^4\,\text{N/m}^2$

(2) $\sigma=\dfrac{M}{I}y=\dfrac{1000}{\dfrac{\pi d^4}{64}}\times\dfrac{d}{2}=127.4\times10^4\,\text{N/m}^2$

(3) $W_C=\dfrac{I}{\dfrac{2}{3}h}=\dfrac{bh^2}{24}=2.0\times10^{-3}\,\text{m}^3$, $W_C=\dfrac{I}{\dfrac{1}{3}h}=\dfrac{bh^2}{12}=4.0\times10^{-3}\,\text{m}^3$

$\therefore\ \sigma_c=-50.0\text{N/m}^2$， $\sigma_t=25.0\text{N/m}^2$

［問題 **9.2**］ $y_0=35\text{mm}$, $I=740\,000\,\text{mm}^4$

$$\sigma_c=-\frac{M}{I}y_C=-\frac{200\,000}{740\,000}\times25=-6.76\,\text{N/mm}^2$$

$$\sigma_t=\frac{M}{I}y_t=\frac{200\,000}{740\,000}\times35=9.46\,\text{N/mm}^2$$

［問題 **9.3**］フランジ部分に対しては

$$G_1=A_1y_1=60\times20\times15=18\,000\,\text{mm}^3,\qquad \tau_1=\frac{QG_1}{bI}=\frac{2\,000\times18\,000}{60\times740\,000}=0.81\,\text{N/mm}^2$$

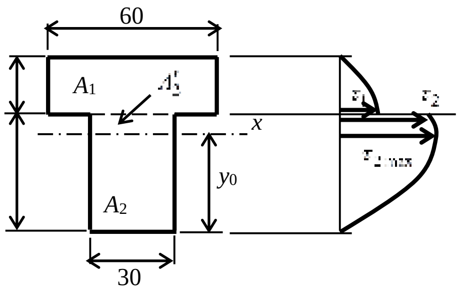
ウエッブ部分に対しては

$$G_2=A_1y_1+A_2'y_2=(60\times20)\times15+(30\times5)\times2.5=18.375\times10^3\,\text{mm}^3$$

$$\tau_2=\frac{QG_1}{bI}=\frac{2\,000\times18.0\times10^3}{30\times740\,000}=1.62\,\text{N/mm}^2$$

$$\tau_{2\max}=\frac{QG_2}{bI}=\frac{2\,000\times18.375\times10^3}{30\times740\,000}=1.66\,\text{N/mm}^2$$

$$\frac{\tau_1}{\tau_2}=\frac{0.81}{1.62}=\frac{1}{2}=\frac{3}{6}$$



y_0=35mm

I=740 000mm^4

10.　モールの応力円

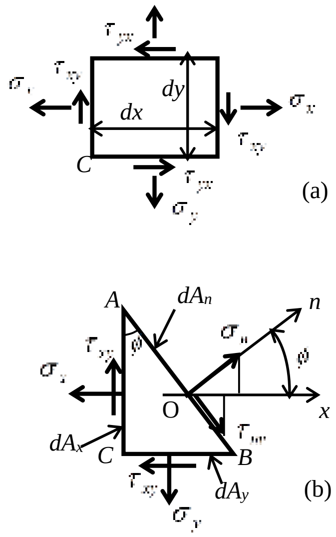


図 4.17

［**問題 10.1**］図 4.17(a)において点 C でモーメント釣合いを考える．奥行き b とする．

$$\sum M_C = 0:\ \tau_{xy}\cdot dy\cdot b\cdot dx - \tau_{yx}\cdot dx\cdot b\cdot dy = 0,\quad \therefore\ \tau_{xy} = \tau_{yx}$$

［**問題 10.2**］各辺の微小面積を dA_x, dA_y, dA_n とすると

$$dA_y = dA_n\cos\phi,\quad dA_x = dA_n\sin\phi$$

$$\sum H = 0:$$

$$\sigma_n\cos\phi\cdot dA_n + \tau_{uv}\sin\phi\cdot dA_n - \sigma_x dA_y + \tau_{xy}dA_x = 0$$

$$\sum V = 0:$$

$$\sigma_n\sin\phi\cdot dA_n - \tau_{uv}\cos\phi\cdot dA_n - \sigma_y dA_x + \tau_{xy}dA_y = 0$$

この 2 式より

$$\left.\begin{array}{l}\sigma_n\cos\phi + \tau_{uv}\sin\phi - \sigma_x\cos\phi + \tau_{xy}\sin\phi = 0\\ \sigma_n\sin\phi - \tau_{uv}\cos\phi - \sigma_y\sin\phi + \tau_{xy}\cos\phi = 0\end{array}\right\}\qquad (1)$$

$(1)_1\times\cos\phi + (1)_2\times\sin\phi$

$$\sigma_n = \sigma_x\cos^2\phi + \sigma_y\sin^2\phi - \tau_{xy}\sin\phi\cos\phi = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\phi - \tau_{xy}\sin 2\phi$$

$(1)_1\times\sin\phi - (1)_2\times\cos\phi$

$$\tau_{uv} = (\sigma_x-\sigma_y)\sin\phi\cos\phi - \tau_{xy}(\sin^2\phi - \cos^2\phi) = \frac{\sigma_x-\sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi$$

［**問題 10.3**］　$\phi = \phi + \pi/2$ を式(4.26)に代入し，$\sin 2\phi = -\sin 2\phi,\ \cos 2\phi = -\cos 2\phi$ を用いる．

［**問題 10.4**］

(1)　$\sigma_u = 16.25\,\mathrm{N/mm^2},\quad \tau_{uv} = 6.49\,\mathrm{N/mm^2}$

(2)　$\sigma_u = 13.75\,\mathrm{N/mm^2},\quad \tau_{uv} = 10.81\,\mathrm{N/mm^2}$

(3)　$\sigma_u = -13.75\,\mathrm{N/mm^2},\quad \tau_{uv} = -10.81\,\mathrm{N/mm^2}$

［**問題 10.5**］図 4.18(b)より

$$\sigma_u = OC + CH\cos(\theta + 2\phi) = OC + CH\cos\theta\cos 2\phi - CH\sin\theta\sin 2\phi$$

$$= \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\phi - \tau_{xy}\sin 2\phi$$

$$\tau_{uv} = CH\sin(\theta + 2\phi) = CH\cos\theta\sin 2\phi - CH\sin\theta\cos 2\phi$$

$$= \frac{\sigma_x-\sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi$$

［**問題 10.6**］問題 4.15 より

$$\sigma_u = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\phi - \tau_{xy}\sin 2\phi = 11.16\,\mathrm{N/mm^2},$$

$$\tau_{uv} = \frac{\sigma_x-\sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi = 12.31\,\mathrm{N/mm^2}$$

式(3.34)より

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = 20.35\,\mathrm{N/mm^2},\ -5.35\,\mathrm{N/mm^2}$$

式(3.35)より　　$\tan 2\alpha = -\dfrac{\tau_{xy}}{\dfrac{\sigma_x-\sigma_y}{2}} = -0.24$　　（モールの応力円は省略）

11.　弾性曲線によるはりのたわみ

［問題 11.1］　2 階の微分方程式より求める．

x 点の曲げモーメントは　$M_x = R_A x - \dfrac{q}{2l}x^2\dfrac{x}{3} = \dfrac{q}{6}(lx - \dfrac{1}{l}x^3)$

これを弾性曲線の微分方程式に代入して積分する．また，簡単のため積分定数は積分の中に含めて計算し，積分記号も省略する．

$$\frac{d^2y}{dx^2} = -\frac{M_x}{EI} = -\frac{q}{6EI}(lx - \frac{1}{l}x^3)$$

$$\theta = \frac{dy}{dx} = -\frac{q}{6EI}\left(\frac{l}{2}x^2 - \frac{1}{4l}x^4 + C_1\right),$$

$$y = -\frac{q}{6EI}\left(\frac{l}{6}x^3 - \frac{1}{20l}x^5 + C_1 x + C_2\right)$$

境界条件を適用する

$x = 0 : y = 0$ より $C_2 = 0,\quad x = l : y = 0$ より $C_1 = -\dfrac{7l^3}{60}$

したがって y, θ は

$$y = -\frac{q}{360EIl}\left(-3x^5 + 10l^2x^3 - 7l^4x\right),\qquad \theta = \frac{dy}{dx} = -\frac{q}{360EIl}\left(-15x^4 + 30l^2x^2 - 7l^4\right)$$

ここで　$\theta_A = \dfrac{7ql^3}{360EI},\quad \theta_B = -\dfrac{ql^3}{45EI}$

最大たわみはたわみ角 $\theta = 0$ より　$15x^4 - 30l^2x^2 + 7l^4 = 0$，これより $x = 0.519l$

これをたわみの式に代入して　$y_{\max} = \dfrac{2.35ql^4}{360EI}$　を得る．

［別解］　4 階の微分方程式より求める．順次積分すると

$$\frac{d^4y}{dx^4} = \frac{q}{EIl}x,\qquad \frac{d^3y}{dx^3} = \frac{q}{2EIl}x^2 + C_1$$

$$\frac{d^2y}{dx^2} = \frac{q}{6EIl}x^3 + C_1x + C_2,\qquad \frac{dy}{dx} = \frac{q}{24EIl}x^4 + \frac{C_1}{2}x^2 + C_2x + C_3$$

$$y = \frac{q}{120EI}x^5 + \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 + C_3x + C_4$$

境界条件を適用する

$x = 0 : y = 0 \to C_4 = 0,\quad x = 0 : M = 0 \to C_2 = 0,\quad x = l : M = 0 \to C_1 = -\dfrac{ql}{6EI},$

$x = l : y = 0 \to C_3 = \dfrac{7ql^3}{360EI}$

これより　$y = -\dfrac{q}{360EIl}\left(-3x^5 + 10l^2x^3 - 7l^4x\right)$　これは上に求めた結果と一致する．

［問題 11.2］　x 点の曲げモーメント：$M_x = \dfrac{M_A}{l}(l - x)$

弾性曲線の微分方程式

$$\frac{d^2y}{dx^2} = -\frac{M_x}{EI} = \frac{M_A}{EIl}(x - l)$$

これを積分する．

$$\theta = \frac{dy}{dx} = \frac{M_A}{EIl}\left(\frac{1}{2}x^2 - lx + C_1\right),\qquad y = \frac{M_A}{EIl}\left(\frac{1}{6}x^3 - \frac{l}{2}x^2 + C_1x + C_2\right)$$

境界条件は　$x=0: y=0 \to C_2=0,\quad x=l: y=0 \to C_1=\dfrac{l^2}{3}$

ゆえに

$$\theta=\frac{M_A}{6EIl}(3x^2-6lx+2l^2),\quad y=\frac{M_A}{6EIl}(x^3-3lx^2+2l^2x)\text{，また，}\quad \theta_A=\frac{M_A l}{3EI},\quad \theta_B=-\frac{M_A l}{6EI}$$

［**問題 11.3**］　荷重より左のたわみを y_1，右のそれを y_2 とする．

$0\le x\le a$

曲げモーメント：$M_x=-\dfrac{M}{l}x$

$$EI\frac{d^2y_1}{dx^2}=\frac{M}{l}x$$

$$EI\frac{dy_1}{dx}=\frac{M}{2l}x^2+C_1$$

$$EIy_1=\frac{M}{6l}x^3+C_1x+C_2$$

$a\le x\le l$

曲げモーメント：$M_x=-\dfrac{M}{l}x+M$

$$EI\frac{d^2y_2}{dx^2}=\frac{M}{l}x-M$$

$$EI\frac{dy_2}{dx}=\frac{M}{2l}x^2-Mx+D_1$$

$$EIy_2=\frac{M}{6l}x^3-\frac{M}{2}x^2+D_1x+D_2$$

境界条件：　$x=0: y_1=0$　　$x=l: y_2=0$

連続条件：　$x=a: y_1=y_2$，$x=a: \theta_1=\theta_2$

この 4 つの条件より　$C_1=\dfrac{M}{6l}(2l^2+3a^2-6la),\quad C_2=0,\quad D_1=\dfrac{M}{6l}(2l^2+3a^2),\quad D_2=-\dfrac{Ma^2}{2}$

ゆえに

$$\theta_1=\frac{M}{6EIl}\left\{3x^2+(2l^2+3a^2-6la)\right\}$$

$$y_1=\frac{M}{6EIl}\left\{x^3+(2l^2+3a^2-6la)x\right\}$$

$$x=0:\theta_A=\frac{M}{6EIl}(2l^2+3a^2-6la)$$

$$a=\frac{l}{2}\ :\ \theta_A=-\frac{Ml}{24EI}$$

$$\theta_2=\frac{M}{6EIl}\left\{3x^2-6lx+(2l^2+3a^2)\right\}$$

$$y_2=\frac{M}{6EIl}\left\{x^3-3lx^2+(2l^2+3a^2)x-3la^2\right\}$$

$$x=l:\theta_B=\frac{M}{6EIl}(-l^2+3a^2)$$

$$a=\frac{l}{2}\ :\ \theta_B=-\frac{Ml}{24EI}=\theta_A$$

$x=a$ におけるたわみ　：$y_{x=a}=\dfrac{Ma}{3EIl}(l^2+2a^2-3la)$

$x=a$ におけるたわみ角：$\theta_{x=a}=\dfrac{M}{3EIl}(l^2+3a^2-3la)$

$a=b=\dfrac{l}{2}$ のとき：$y_{\frac{l}{2}}=0,\quad \theta_{\frac{l}{2}}=\dfrac{Ml^2}{12EIl}$

［**問題 11.4**］　x 点の曲げモーメント　$M_x=M$

弾性曲線の微分方程式：$\dfrac{d^2y}{dx^2}=-\dfrac{M}{EI}$

これを積分する．

$$\theta=\frac{dy}{dx}=-\frac{M}{EI}(x+C_1),\quad y=-\frac{M}{EI}\left(\frac{1}{2}x^2+C_1x+C_2\right)$$

境界条件　$x=0: y=0,\ \theta=0$　より　$C_1=C_2=0$

したがって　$\theta=-\dfrac{M}{EI}x,\quad y=-\dfrac{M}{2EI}x^2$

［**問題 11.5**］　反力は　$R_A = -\dfrac{Pa}{l}$,　$R_B = \dfrac{P(l+a)}{l}$

$0 \le x \le l$

曲げモーメント：$M_x = -\dfrac{Pa}{l}x$

$$EI\frac{d^2y_1}{dx^2} = \frac{Pa}{l}x$$

$$EI\frac{dy_1}{dx} = \frac{Pa}{2l}x^2 + C_1$$

$$EIy_1 = \frac{Pa}{6l}x^3 + C_1x + C_2$$

境界条件：　$x=0 : y_1 = 0$

$x=l : y_1 = 0$

$l \le x \le l+a$

曲げモーメント：$M_x = -\dfrac{Pa}{l}x + \dfrac{P(l+a)}{l}(x-l)$

$$EI\frac{d^2y_2}{dx^2} = \frac{Pa}{l}x - \frac{P(l+a)}{l}(x-l)$$

$$EI\frac{dy_2}{dx} = \frac{Pa}{2l}x^2 - \frac{P(l+a)}{2l}(x-l)^2 + D_1$$

$$EIy_2 = \frac{Pa}{6l}x^3 - \frac{P(l+a)}{6l}(x-l)^3 + D_1x + D_2$$

$x=l : y_2 = 0$

連続条件：　$x=l : \theta_1 = \theta_2$

この 4 つの条件より　$C_1 = D_1 = -\dfrac{P\,a\,l}{6}$,　$C_2 = D_2 = 0$

ゆえに

$$\theta_1 = \frac{Pa}{6EIl}(3x^2 - l^2)$$

$$y_1 = \frac{Pa}{6EIl}(x^3 - l^2x)$$

$x=0 : \theta_A = -\dfrac{Pal}{6EI}$,　$x=l : \theta_{1B} = \dfrac{Pal}{3EI}$

$$\theta_2 = \frac{P}{6EIl}\left\{3ax^2 - 3(l+a)(x-l)^2 - al^2\right\}$$

$$y_2 = \frac{P}{6EIl}\left\{ax^3 - (l+a)(x-l)^3 - al^2x\right\}$$

$x=l : \theta_{2B} = \dfrac{Pal}{3EI} = \theta_{1B}$

$x=l+a : \theta_C = \dfrac{Pa}{6EI}(2l+3a)$,　$y_C = \dfrac{Pa^2}{3EI}(l+a)$

［**問題 11.6**］（解略）例題 5.1，5.5，問題 5.4 参照．**(2)**の点 A のたわみ角は負になることに注意.

12.　弾性荷重法によるはりのたわみ

［問題 12.1］

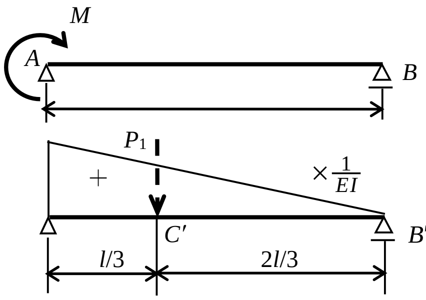


図 21.8

共役ばりにおいて

$$P_1 = \frac{Ml}{2}$$

が三角形の図心位置に作用するとして，反力は

$$R'_A = \frac{Ml}{2}\frac{2l}{3}\frac{1}{l} = \frac{Ml}{3}$$

$$R'_B = \frac{Ml}{2}\frac{l}{3}\frac{1}{l} = \frac{Ml}{6}$$

ゆえに

$$\theta_A = \frac{Q_{A'}}{EI} = \frac{Ml}{3EI}$$

$$\theta_B = \frac{Q_{B'}}{EI} = -\frac{Ml}{6EI}$$

［問題 12.2］

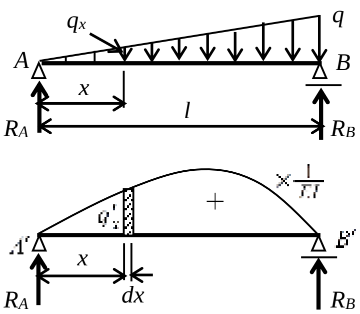


図 21.94

反力は　$R_A = \frac{ql}{6}$，$R_b = \frac{ql}{3}$

弾性荷重の荷重強度 q'_x は

$$q'_x = \frac{q}{6l}(l^2 x - x^3)$$

弾性荷重の大きさは

$$A = \int_0^l q'_x dx = \frac{q}{6l}\int_0^l (l^2 x - x^3)dx$$

$$= \frac{q}{6l}\left[\frac{l^2}{2}x^2 - \frac{1}{4}x^4\right]_0^l = \frac{ql^3}{24}$$

点 A を通る鉛直軸に対する断面 1 次モーメントは

$$G_x = \int_0^l x dA = \int_0^l x \cdot q'_x dx = \frac{q}{6l}\int_0^l (l^2 x^2 - x^4)dx = \frac{q}{6l}\left[\frac{l^2}{3}x^3 - \frac{1}{5}x^5\right]_0^l = \frac{ql^4}{45}$$

ゆえに，弾性荷重の図心位置は

$$x_0 = \frac{G_x}{A} = \frac{ql^4}{45}\bigg/\frac{ql^3}{24} = \frac{8}{15}l$$

共役ばりの反力は

$$R_{A'} = \frac{1}{l}\cdot\frac{ql^3}{24}\cdot\frac{7}{15}l = \frac{7ql^3}{360},\quad R_{B'} = \frac{1}{l}\cdot\frac{ql^3}{24}\cdot\frac{8}{15}l = \frac{8ql^3}{360}$$

ゆえに

$$\theta_A = \frac{R_{A'}}{EI} = \frac{7ql^3}{360EI},\quad \theta_B = \frac{R_{B'}}{EI} = -\frac{8ql^3}{360EI}$$

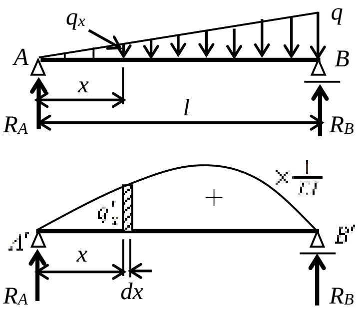
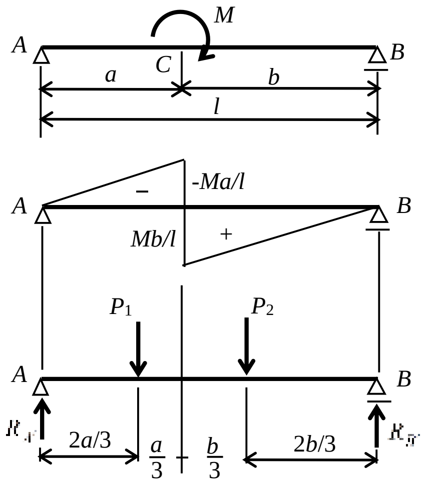


図 21.94

［問題 12.3］



$$R_A=-\frac{M}{l},\ \ R_B=\frac{M}{l}$$

$$P_1=-\frac{Ma^2}{2l},\quad P_2=\frac{Mb^2}{2l}$$

$$R_{A'}=\frac{M}{6l^2}\left(-a^3-3a^2b+2b^3\right),$$

$$R_{B'}=\frac{M}{6l^2}\left(-2a^3+3ab^2+b^3\right)$$

$$Q_{C'}=R_{A'}-P_1=\frac{M}{3l^2}(a^3+b^3)$$

$$M_{C'}=R_{A'}a-P_1\frac{a}{3}=\frac{Mab}{3l^2}(-a^2+b^2)$$

図 21.10

以上より

$$\theta_A=\frac{Q_{A'}}{EI}=\frac{M}{6EIl^2}(-a^3-3a^2b+2b^3),\qquad \theta_B=\frac{Q_{B'}}{EI}=-\frac{M}{6EIl^2}(-2a^3+3ab^2+b^3)$$

$$\theta_C=\frac{Q_{C'}}{EI}=\frac{M}{3EIl^2}(a^3+b^3),\qquad y_C=\frac{M_{C'}}{EI}=\frac{Mab}{3EIl^2}(-a^2+b^2)$$

［問題 12.4］

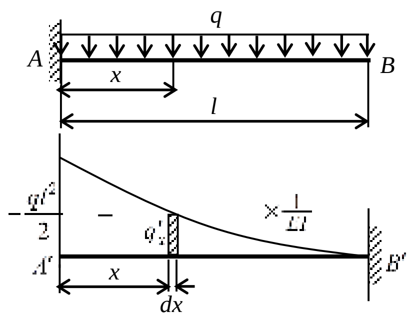


図 21.11

x 点の曲げモーメントは

$$q'_x=-\frac{q}{2}(x-l)^2$$

点 A から x の点の微小面積について考える．

$$R_{B'}=\int_0^l q'_x dx=-\frac{q}{2}\int_0^l (x-l)^2 dx=-\frac{ql^3}{6}$$

$$Q_{B'}=-R_{B'}=\frac{ql^3}{6},\qquad \therefore\ \theta_B=\frac{Q_{B'}}{EI}=\frac{ql^3}{6EI}$$

$$M_{B'}=\int_0^l (q'_x dx)(l-x)=\frac{q}{2}\int_0^l (x-l)^3 dx=\frac{ql^4}{8}$$

$$\therefore\ y_B=\frac{M_{B'}}{EI}=\frac{ql^4}{8EI}$$

［問題 12.5］

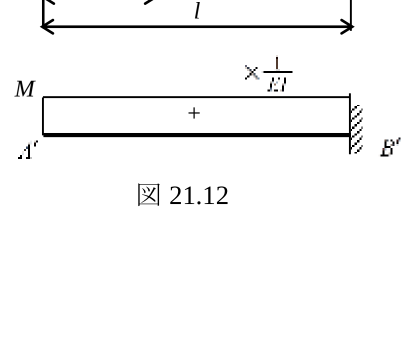


図 21.12

曲げモーメントは正で一定である．

$$R_{B'}=Ml,\ \ \theta_B=\frac{Q_{B'}}{EI}=-\frac{Ml}{EI}$$

$$M_{B'}=-Ml\cdot\frac{l}{2},\quad y_B=\frac{M_{B'}}{EI}=-\frac{Ml^2}{2EI}$$

［**問題 12.6**］

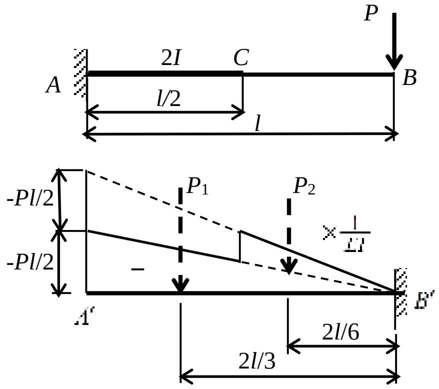


図 21.13

弾性荷重を 1/EI で統一するために，断面 2 次モーメントが 2I の部分の曲げモーメントを 1/2 倍する．

$$P_1 = -\frac{Pl^2}{4}, \quad P_2 = -\frac{Pl^2}{16}$$

$$R_{B'} = P_1 + P_2 = -\frac{5Pl^2}{16}, \quad \therefore \theta_B = \frac{Q_{B'}}{EI} = \frac{5Pl^2}{16EI}$$

$$M_{B'} = -P_1\frac{2l}{3} - P_2\frac{l}{3} = \frac{3Pl^3}{16}, \quad \therefore \quad y_B = \frac{M_{B'}}{EI} = \frac{3Pl^3}{16EI}$$

［**問題 12.7**］

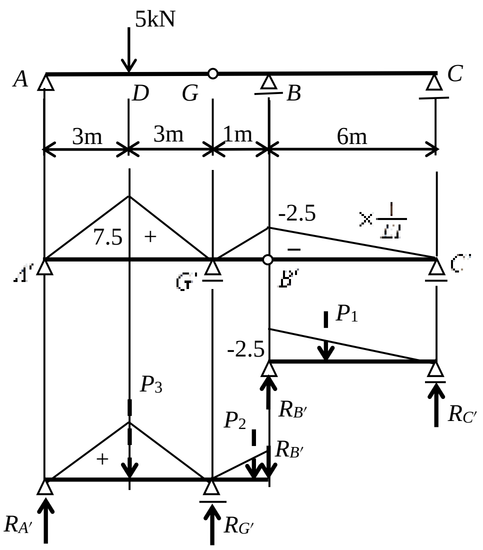


図 21.14

単位は[kN], [kNm]とする．

$$P_1 = -7.5, \quad P_2 = -1.25, \quad P_3 = 22.5$$

$$R_{B'} = -5, \quad R_{C'} = -2.5$$

$$\therefore \quad \theta_B = -\frac{5}{EI}, \quad \theta_C = \frac{2.5}{EI}$$

$$\sum M_{G'} = 0 : R_{A'} \cdot 6 - 22.5 \cdot 3 + (-1.25) \cdot \frac{2}{3} + R_{B'} \cdot 1 = 0$$

$$\therefore R_{A'} = 12.22, \quad \therefore \quad \theta_A = \frac{Q_{A'}}{EI} = \frac{12.22}{EI}$$

$$M_{G'} = -P_2 \cdot \frac{2}{3} - R_{B'} \cdot 1 = 1.25 \cdot \frac{2}{3} + 5 = \frac{2.5}{3} = 5.83$$

$$\therefore \quad y_G = \frac{M_{G'}}{EI} = \frac{5.83}{EI}$$

$${}_rQ_{G'} = P_2 + R_{B'} = -6.25, \quad {}_lQ_{G'} = R_{A'} - P_3 = -10.28$$

$$\therefore \quad {}_r\theta_G = -\frac{6.25}{EI}, \quad {}_l\theta_G = -\frac{10.28}{EI}$$

18　静定トラス

［問題 18.1］

(1)　$\cos\theta = \dfrac{4}{5},\ \sin\theta = \dfrac{3}{5},$

$$N_{AC}\sin\theta + P = 0,\quad \therefore N_{AC} = -\frac{5}{3}P,\quad N_{AC}\cos\theta + N_{AB} = 0,\quad \therefore N_{AB} = -N_{AC}\cos\theta = \frac{4}{3}P$$

(2)　$\cos\theta = \sin\theta = \dfrac{1}{\sqrt{2}}$

$$N_{CB} = 0,\quad N_{CD} = -P,\quad N_{BD}\sin\theta + N_{CD} = 0,\quad \therefore N_{BD} = \sqrt{2}P$$

$$N_{BD}\cos\theta + N_{AD} = 0,\quad \therefore N_{AD} = -N_{BD}\cos\theta = -P$$

(3)　$\sum M_A = 0: -H_D a + Pa = 0,\ \therefore\ H_D = P,\quad \sum M_D = 0: V_A a - H_A a = 0,\ \therefore\ V_A = H_A = P$

$$N_{BD} = 0,\quad N_{CD} = -P,\quad N_{AC} = -P,\quad N_{AB} = -P$$

$$N_{CB}\cos\theta + N_{CD} = 0,\quad \therefore\ N_{CB} = -\frac{N_{CD}}{\cos\theta} = \sqrt{2}P$$

(4)　$\sum M_B = 0: R_A a + P\cdot 2a = 0,\ \therefore\ R_A = -2P,\ R_B = 2P,\quad H_A = -P$

$$N_{DE}\sin\theta + P = 0,\ \therefore\ N_{DE} = -\frac{P}{\sin\theta} = -\sqrt{2}P,\quad N_{DE}\cos\theta + N_{CE} = 0,\quad \therefore\ N_{CE} = -N_{DE}\cos\theta = P$$

$$N_{CD} = 0,\quad N_{AC} = N_{CE} = P,\quad N_{AB} = 0,\quad N_{BD} = -2P$$

$$H_A + N_{AD}\cos\theta = 0,\quad \therefore\ N_{AD} = -\frac{H_A}{\cos\theta} = \sqrt{2}P$$

(5)　$\sin\theta = \dfrac{4}{5},\ \cos\theta = \dfrac{3}{5}$

反力

$$\sum M_B = 0: R_A\cdot 12 - 2\cdot 9 - 4\cdot 6 - 5\cdot 3 = 0,\ \therefore\ R_A = \frac{57}{12} = 4.75\text{kN},\quad R_B = 2 + 4 + 5 - \frac{57}{12} = \frac{75}{12} = 6.25\text{kN}$$

部材力

A: $D_1\sin\theta + R_A = 0,\ \therefore\ D_1 = -5.94\text{kN},\quad D_1\cos\theta + L_1 = 0,\ \therefore\ L_1 = 3.56\text{kN}$

1: $D_1\sin\theta + D_2\sin\theta + 2 = 0,\ \therefore\ D_2 = 3.44\text{kN},\quad -D_1\cos\theta + D_2\sin\theta + U_1 = 0,\ \therefore\ U_1 = -5.63\text{k}N$

2: $D_2\sin\theta + D_3\sin\theta - 4 = 0,\ \therefore\ D_3 = 1.56\text{kN}$

B: $D_4\sin\theta + R_B = 0,\ \therefore\ D_4 = -7.81\text{kN},\quad D_4\cos\theta + L_2 = 0,\ \therefore\ L_2 = 4.69\text{kN}$

(6)　$\sin\theta = \dfrac{4}{5},\ \cos\theta = \dfrac{3}{5}$

反力

$$\sum M_B = 0: R_A\cdot 9 - 10\cdot 6 - 5\cdot 3 = 0,\ \therefore\ R_A = \frac{25}{3} = 8.33\text{kN},\quad R_B = 10 + 5 + \frac{25}{3} = \frac{20}{3} = 6.67\text{kN}$$

部材力

A: $D_1\sin\theta + R_A = 0,\ \therefore\ D_1 = -10.41\text{kN},\quad D_1\cos\theta + L_1 = 0,\ \therefore\ L_1 = 6.25\text{kN}$

1′: $D_1\sin\theta + V_1 = 0,\ \therefore\ V_1 = 8.33\text{kN},\quad -D_1\cos\theta + U_1 = 0,\ \therefore\ U_1 = -6.25\text{kN}$

2: $R_A + D_2\sin\theta - 10,\ \therefore D_2 = 2.09kN$

B: $D_3\sin\theta + R_B = 0,\ \therefore\ D_3 = -8.33\text{kN},\quad D_3\cos\theta + L_B = 0,\ \therefore\ L_3 = 5.00\text{kN}$

2: $V_2 = 0,\ \ L_2 = L_3 = 5.00\text{kN}$

［問題 18.2］

(1) $\cos\theta = \sin\theta = \dfrac{1}{\sqrt{2}}$

$$\sum M_B = 0: R_A \cdot 4a - P\cdot 3a - 2P\cdot 2a - 4P\cdot a = 0,\quad \therefore\ R_A = \frac{11P}{4} = 2.75P,\qquad R_B = \frac{17}{4}P$$

$$U_2 = -\frac{M_2'}{a} = -\frac{R_A\cdot 2a - Pa}{a} = -\frac{18}{4}P = -4.5P,\qquad D_2 = \frac{Q_{1-2}}{\sin\theta} = \frac{R_A - P}{\sin\theta} = \frac{7\sqrt{2}}{4}P = 2.47P,$$

$$L_1 = \frac{M_1}{a} = \frac{11}{4}P = 2.75P,\qquad V_1 = -Q_{A-1} = -R_A + P = -\frac{7}{4}P = -1.75P$$

(2) $\sin\theta = \dfrac{4}{5},\ \cos\theta = \dfrac{3}{5}$

$$\sum M_B = 0: R_A\cdot 6 - 10\cdot 4 = 0,\quad \therefore\ R_A = \frac{40}{6} = 6.67\text{kN},\qquad R_B = -\frac{40}{6} = -6.67\text{kN},\qquad H_A = 10\text{kN}$$

$$U_2 = -10\text{kN},\qquad V_2 = 0,\qquad D_2\sin\theta + V_2 + R_B = 0,\ \therefore D_2 = -\frac{R_B}{\sin\theta} = \frac{25}{3} = 8.33\text{kN},$$

$$D_2\cos\theta + L_1 = 0,\quad \therefore\ L_1 = -D_2\cos\theta = -5.00\text{kN}$$

(3)

$$\sum M_B = 0: R_A\cdot 12 - 4\cdot 12 - 10\cdot 9 - 4\cdot 3 = 0,\quad \therefore\ R_A = \frac{150}{12} = 12.5kN,\qquad R_B = 4 + 10 + 4 - 12.5 = 5.5kN$$

$$U_2 = -\frac{M_2}{h} = -\frac{(R_A - 4)\cdot 6 + 10\cdot 3}{4} = -\frac{42}{8} = -5.25\text{kN},\qquad D_2 = \frac{Q_{1-2}}{\sin\theta} = -\frac{15}{8} = -1.875\text{kN},$$

$$L_2 = \frac{M_{1'}}{h} = \frac{(R_A - 4)\cdot 3}{4} = \frac{51}{8} = 6.375\text{kN},\qquad V_2 = -Q_{1-2} = -(R_A - 4) = -\frac{17}{2} = -8.5\text{kN}$$

(4) $\sin\theta = \cos\theta = \dfrac{1}{\sqrt{2}}$

$$R_A = R_B = 15kN$$

$$A:\ U_1\sin\theta + R_A = 0,\quad \therefore\ U_1 = -15\sqrt{2} = \text{-21.15kN}$$

$$U_1\cos\theta + L_1 = 0,\quad \therefore\ L_1 = -U_1\cos\theta = 15\text{kN}$$

$$D:\ D_1 = -5\sqrt{2} = \text{-7.07kN},\qquad -U_1 - 5\sqrt{2} + U_2 = 0,\quad \therefore\ U_2 = U_1 + 5\sqrt{2} = -10\sqrt{2} = \text{-14.14kN}$$

$$F:\ U_2\sin\theta + U_2'\sin\theta + V_1 + 10 = 0,\quad \therefore\ V_1 = 10\text{kN}$$

(5) 三角形の相似より

$$R_A = R_B = \frac{3}{2}P$$

$$\Delta 1'2'M \backsim \Delta 22'N\ \text{より}\ 1'2' : 1'M = 2'2 : 2N,\ \therefore\ 2N = h = \frac{1'M\times 2'2}{1'2'} = \frac{4\times 5}{\sqrt{4^2+1^2}} = \frac{20}{\sqrt{17}} = 4.85\text{m}$$

$$\Delta 1'\text{O}1 \backsim \Delta 2'1'\text{M}\ \text{より}\ x : 4 = 4 : 1,\ \therefore\ x = \frac{16}{1} = 16\text{m},\quad \therefore\ a = 12\text{m}$$

$$\Delta 2\text{O}P \backsim \Delta 21'1\ \text{より}\ OP : O2 = 1'1 : 1'2,\ \therefore\ \text{O}P = b = \frac{20\times 4}{4\sqrt{2}} = \frac{20}{\sqrt{2}} = 10\sqrt{2} = 14.14\text{m}$$

$$\sum M_2 = 0:\ R_A\cdot 2\lambda - P\cdot\lambda + U_2\cdot h = 0,\quad \therefore U_2 = -\frac{R_A\cdot 2\lambda - P\cdot\lambda}{h} = -\frac{8P}{4.85} = -1.65P$$

$$\sum M_1' = 0:\ R_A\cdot\lambda - L_2\cdot h_1 = 0,\quad \therefore L_2 = -\frac{R_A\cdot\lambda}{h_1} = 1.5P$$

$$\sum M_\text{O} = 0:\ -R_A\cdot a + P(a+\lambda) + D_2\cdot b = 0,\quad \therefore D_2 = \frac{R_A\cdot a - P(a+\lambda)}{b} = \frac{\sqrt{2}P}{10} = 0.14P$$

(6)

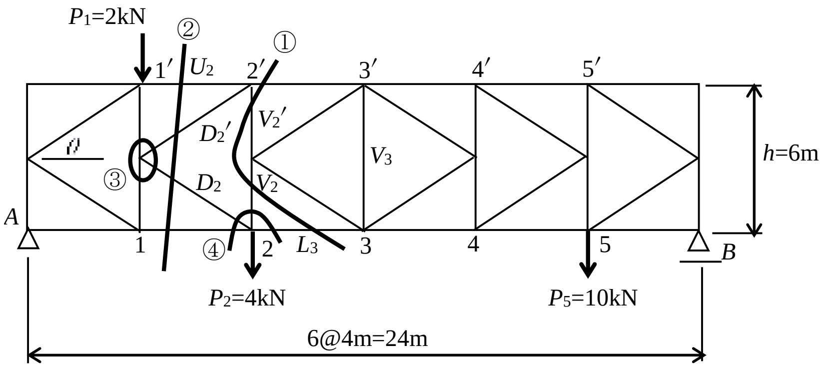


図 8.19

$$R_A \cdot 24 - 2\times 20 - 4\times 16 - 10\times 4 = 0, \quad \therefore\ R_A = 6\text{kN}, \quad R_B = 2+4+10-6 = 10\text{kN}$$

① :　$\sum M_{2'} = 0 : R_A \cdot 8 - 2\cdot 4 - L_3 \cdot 6 = 0, \quad L_3 = \dfrac{20}{3} = 6.67\text{kN}$

③ :　$D_2 = -D_2'$

② :　$\sum V = 0 :\ R_A - 2 + (D_2' - D_2)\sin\theta = 0, \quad \therefore\ D_2' = -\dfrac{R_A - 2}{2\sin\theta} = -\dfrac{10}{3} = -3.33\text{kN}$

$\sum M_1 = 0 :\quad R_A \cdot 4 - U_2 \cdot 6 = 0, \quad U_2 = -4\text{kN}$

④ :　$\sum V = 0 :\ D_2 \sin\theta + V_2 - 4 = 0, \quad \therefore\ V_2 = 4 - D_2 \sin\theta = 2\text{kN}$

$\sum V = 0 :\ D_2' \sin\theta + V_2' = 0, \quad \therefore\ V_2' = -D_2' \sin\theta = 2\text{kN}$

$V_3 = 0$

(7)

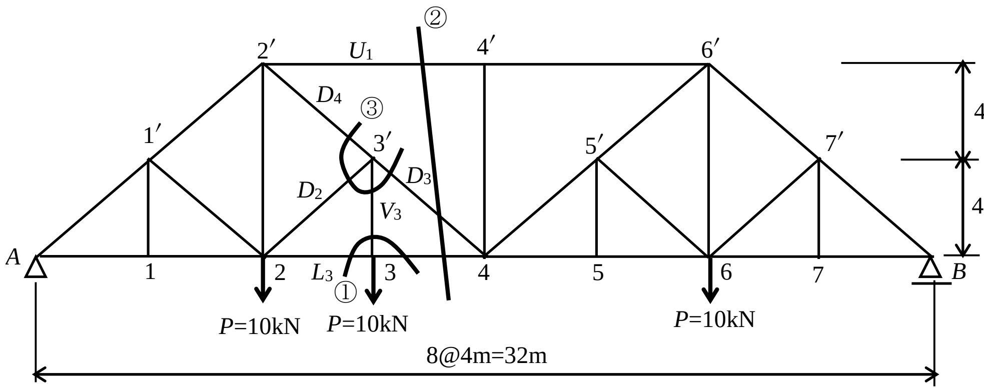


図 8.20

$$R_A \cdot 32 - 10\times 24 - 10\times 20 - 10\times 8 = 0, \quad \therefore\ R_A = \frac{65}{4} = 16.25\text{kN}, \quad R_B = 13.75\text{kN}$$

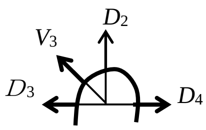
① :　$V_3 = 10\text{kN}$

② :　$\sum M_4 = 0 :\ R_A \cdot 16 - 10\times 8 - 10\times 4 + U_1 \times 8 = 0, \quad \therefore U_1 = -\dfrac{35}{2} = -17.5\text{kN}$

$\sum V = 0 :\quad R_A - 10 - 10 - D_3 \sin\theta = 0, \quad \therefore\ D_3 = -\dfrac{15\sqrt{2}}{4} = -3.75\sqrt{2} = \text{-}5.29\text{kN}$

③ :　$\sum V = 0 :\quad V_3 \sin\theta + D_2 = 0, \quad \therefore\ D_2 = -5\sqrt{2} = -7.07\text{kN}$

$\sum H = 0 :\quad D_3 + V_3 \cos\theta - D_4 = 0, \quad \therefore\ D_4 = \dfrac{5\sqrt{2}}{4} = 1.25\sqrt{2} = 1.77\text{kN}$

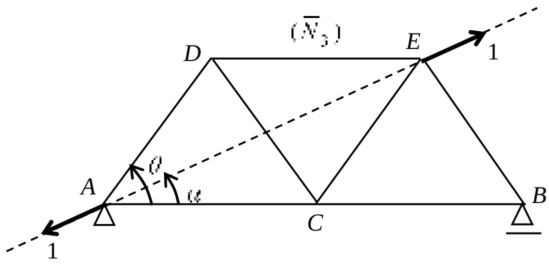


②′ :　$\sum M_{2'} = 0 :\ R_A \cdot 8 + 10\cdot 4 - L_4 \cdot 8 = 0, \quad \therefore\ L_4 = \dfrac{1}{8}(R_A \cdot 8 + 10\cdot 4) = 21.25\text{kN} = L_3$

［**問題 18.3**］図**(1),(2),(3)**の部材力（$N, \overline{N}_1, \overline{N}_2$）を下表に記す.

部材	l	N	$\overline{N}_1$	$\overline{N}_2$
1	6	6.00	0.375	1
2	6	9.75	0.375	0
3	5	-10.0	-0.625	0
4	5	7.50	0.625	0
5	5	1.25	0.625	0
6	5	-16.25	-0.625	0
7	6	-10.50	-0.750	0

(4)



$$R_A = R_B = 0,\quad \sin\theta = 4/5 = 0.8,\quad \cos\theta = 3/5 = 0.6,\quad \sin\alpha = 4/\sqrt{97} = 0.406,\quad \cos\alpha = 9/\sqrt{97} = 0.914$$

1)　節点 A：　$R_A + N_3\sin\theta - 1\cdot\sin\alpha = 0,\quad \therefore\ N_3 = \dfrac{\sin\alpha}{\sin\theta} = \dfrac{5}{\sqrt{97}} = 0.508$

$N_1 + N_3\cos\theta - \cos\alpha = 0,\quad \therefore\ N_1 = \cos\alpha - N_3\cos\theta = \dfrac{6}{\sqrt{97}} = 0.609$

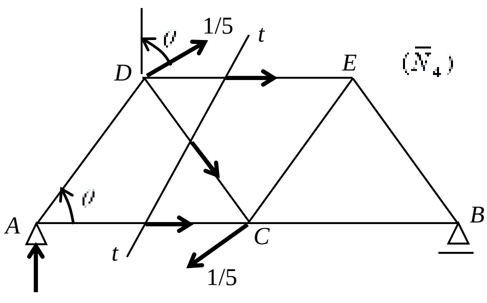
2)　節点 B：　$N_3\sin\theta + N_4\sin\alpha = 0,\quad \therefore\ N_4 = -N_3 = -\dfrac{5}{\sqrt{97}} = -0.508$

$N_7 + N_4\cos\theta - N_3\cos\theta = 0,\quad \therefore\ N_7 = (N_3 - N_4)\cos\theta = \dfrac{6}{\sqrt{97}} = 0.609$

3)　節点 C：　$N_4\sin\theta + N_5\sin\theta = 0,\quad \therefore\ N_5 = -N_4 = \dfrac{5}{\sqrt{97}} = 0.508$

$N_6 = 0,\quad N_2 = 0$

(5)



$$R_A = -R_B = -1/12 = -0.083,\quad \sin\theta = 4/5 = 0.8,\quad \cos\theta = 3/5 = 0.6$$

1)　節点 A：　$R_A + N_3\sin\theta = 0,\quad \therefore\ N_3 = -\dfrac{R_A}{\sin\theta} = \dfrac{5}{48} = 0.104$

$N_1 + N_3\cos\theta = 0,\quad \therefore\ N_1 = -N_3\cos\theta = -\dfrac{3}{48} = -0.0625$

2)　断面 t-t　$\sum M_C = 0:\ 6R_A + 5\cdot\dfrac{1}{5} + 4N_7 = 0,\quad \therefore\ N_7 = -\dfrac{1}{8} = -0.125$

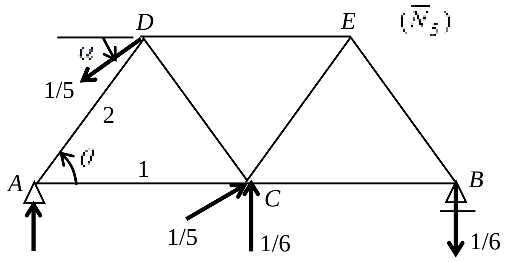
$$\sum V = 0: \ R_A + \frac{1}{5}\cos\theta - N_4 \sin\theta = 0, \quad \therefore \ N_4 = \frac{11}{240} = 0.0458$$

3)　節点 B：　$R_B + N_6 \sin\theta = 0, \quad \therefore \ N_6 = -\dfrac{R_B}{\sin\theta} = -\dfrac{5}{48} = -0.104$

$$N_2 + N_6 \cos\theta = 0, \quad \therefore \ N_2 = -N_6\cos\theta = -\frac{3}{48} = -0.0625$$

4)　節点 E：　$N_5 \sin\theta + N_6 \sin\theta = 0, \quad \therefore \ N_5 = -N_6 = \dfrac{5}{48} = 0.104$

(6)



$R_A = R_B = 0, \ \sin\theta = 4/5 = 0.8, \quad \cos\theta = 3/5 = 0.6$

$\cos\alpha = \cos(90-\theta) = \sin\theta = 4/5, \quad \sin\alpha = \sin(90-\theta) = \cos\theta = 3/4$

1)　$N_1 = N_2 = 0$

2)　節点 B：　$-\dfrac{1}{6} + N_6 \sin\theta = 0, \quad \therefore \ N_6 = \dfrac{1/6}{\sin\theta} = \dfrac{5}{24} = 0.208$

$$N_2 + N_6 \cos\theta = 0, \quad \therefore \ N_2 = -N_6 \cos\theta = -\frac{1}{8} = -0.125$$

3)　節点 E：　$N_5 \sin\theta + N_6 \sin\theta = 0, \quad \therefore \ N_5 = -N_6 = -\dfrac{5}{48} = -0.208$

$$N_7 + N_5 \cos\theta - N_6 \cos\theta = 0, \quad \therefore \ N_7 = (N_6 - N_5)\cos\theta = \frac{1}{4} = 0.250$$

4)　節点 C：　$N_4 \sin\theta + N_5 \sin\theta + \dfrac{1}{5}\sin\alpha + \dfrac{1}{6} = 0, \quad \therefore \ N_4 = -\dfrac{5}{4}\left(N_5 \dfrac{4}{5} + \dfrac{1}{5}\cdot\dfrac{3}{5} + \dfrac{1}{6}\right) = -\dfrac{3}{20} = -0.150$

以上をまとめる.

部材	l	$\overline{N}_3$	$\overline{N}_4$	$\overline{N}_5$
1	6	0.609	-0.063	0.0
2	6	0.0	0.063	-0.125
3	5	0.508	0.104	0.0
4	5	-0.508	0.046	-0.150
5	5	0.508	0.104	-0.208
6	5	0.0	-0.104	0.208
7	6	0.609	-0.125	0.250

20.　トラスの影響線

［問題 20.1］

(1)　ハウトラスの影響線.

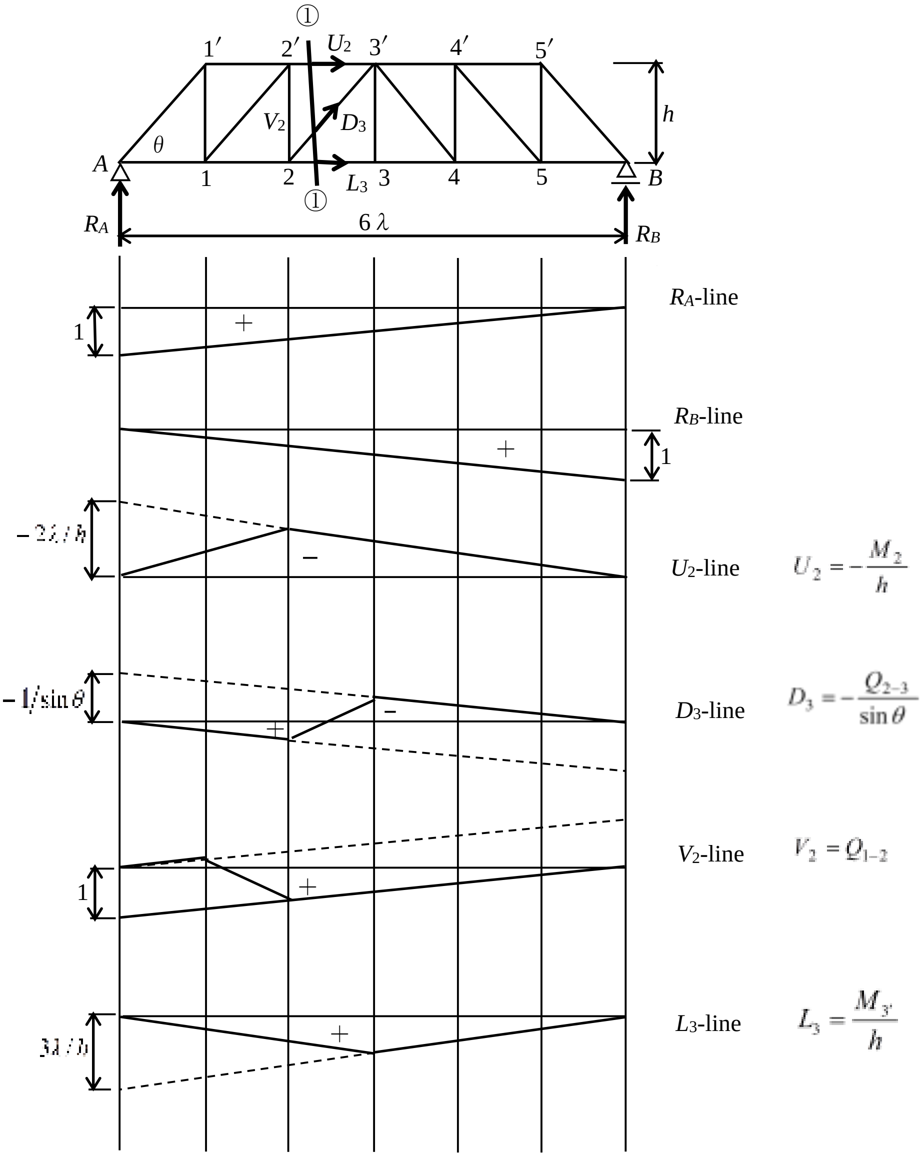


図 8.24

(2)　プラットトラスの影響線.

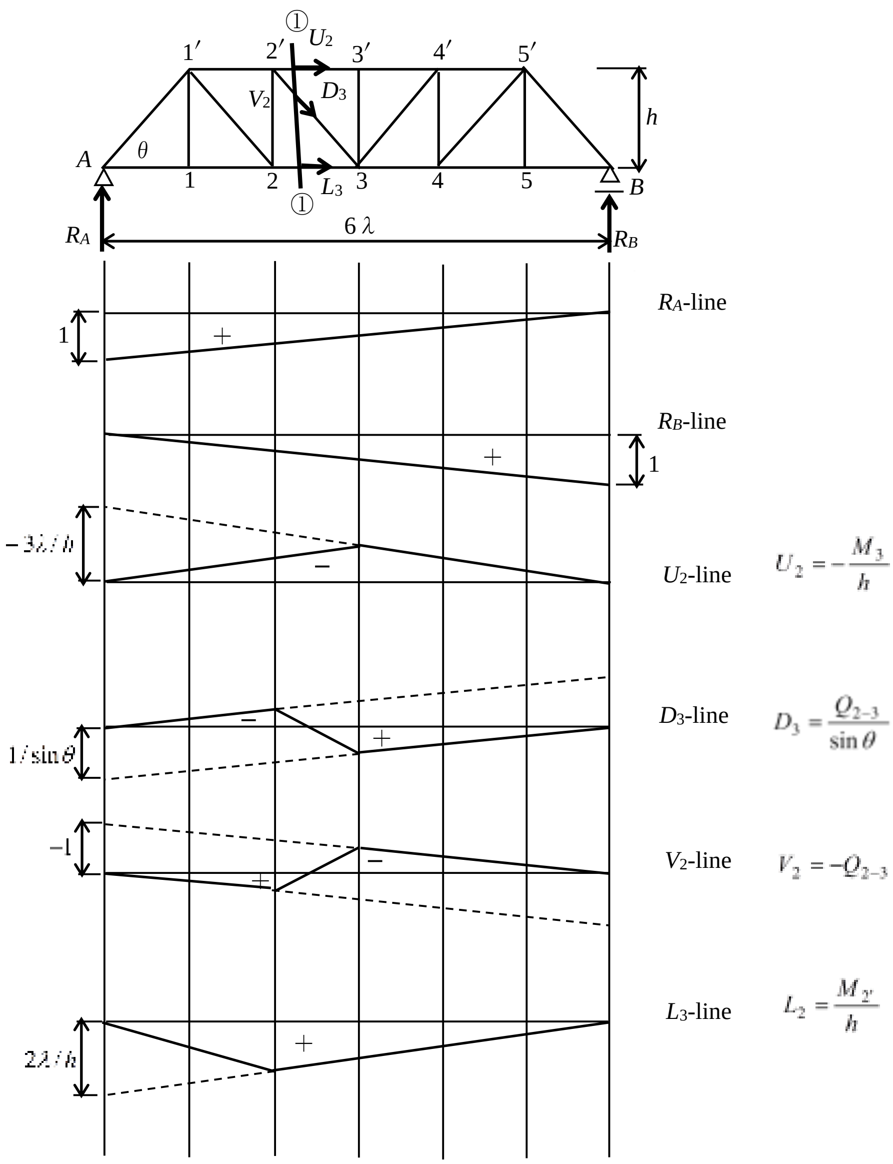


図 8.25

(3)　ワーレントラスの影響線.

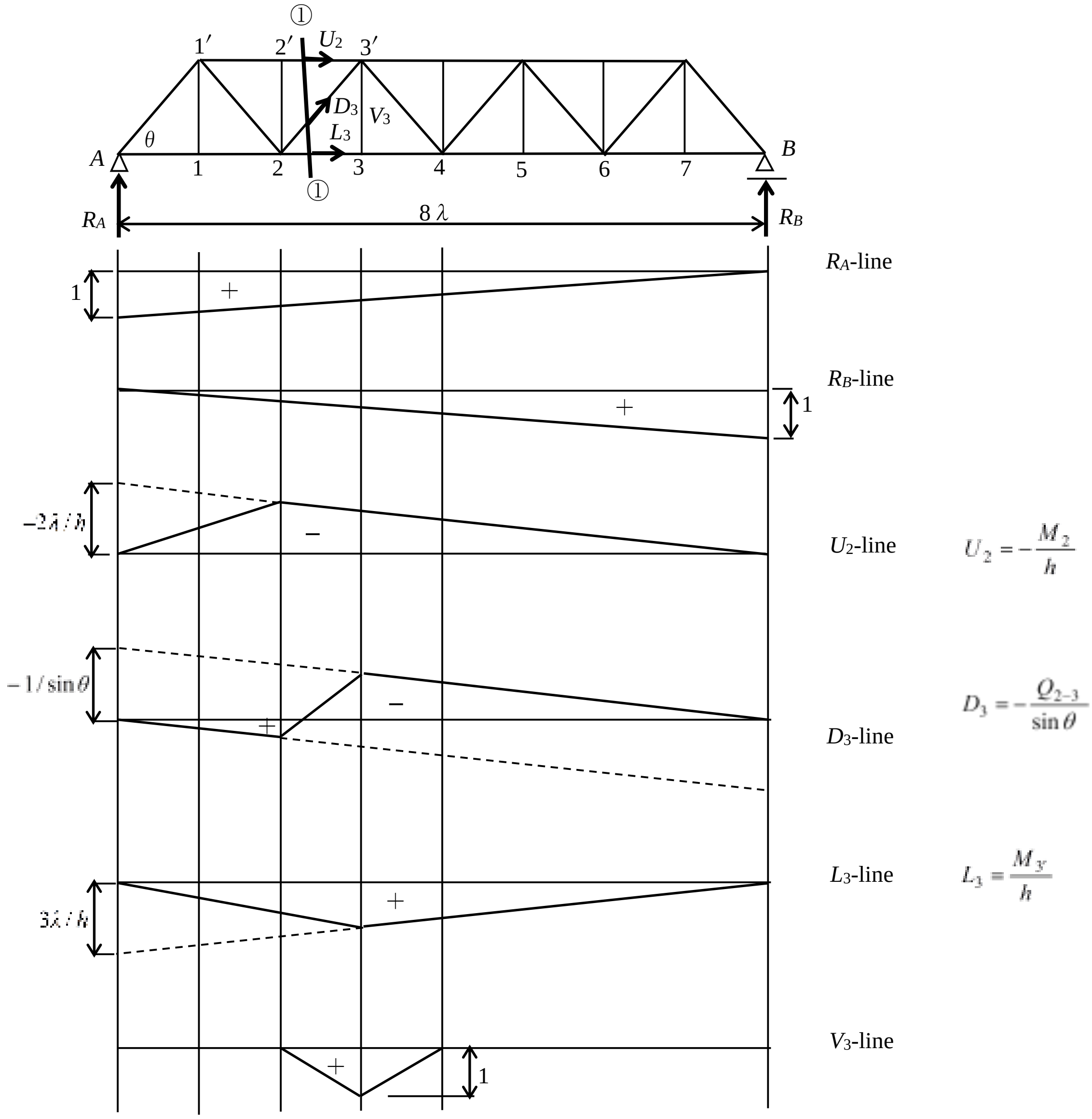


図 8.26

(4)　曲弦トラスの影響線.

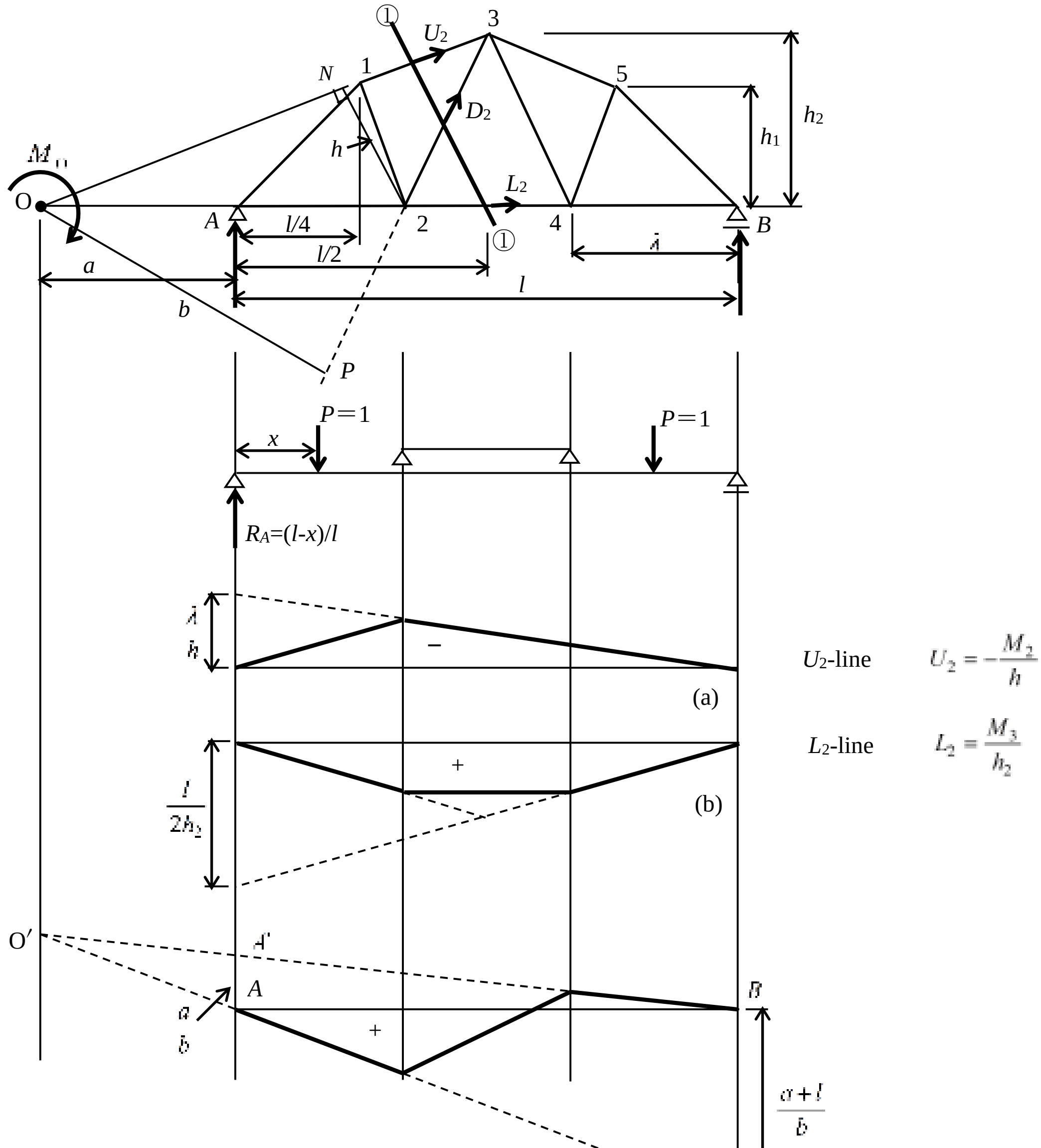


図 8.27

D_2-line

$2\lambda \le x \le l$

$$\sum M_O = 0$$

$$-R_A a - D_2 b = 0, \quad \therefore\ D_2 = -\frac{a}{b} R_A = -\frac{a}{b}\frac{l-x}{l}$$

$$x = 0 : D_2 = -\frac{a}{b}, \quad x = l : D_2 = 0$$

$0 \le x \le \lambda$

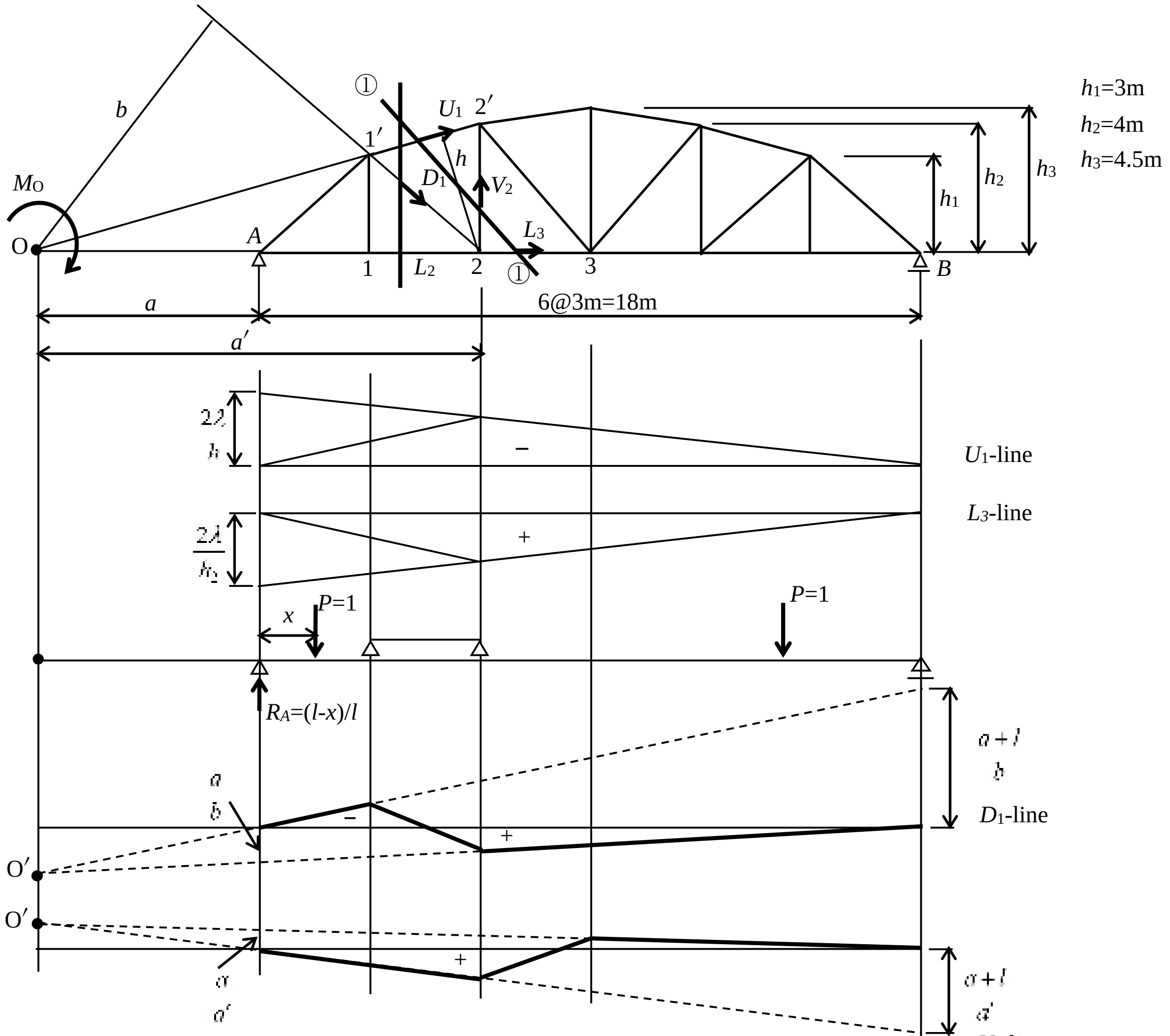
$$\sum M_O = 0$$

$$-R_A a + 1\cdot(a+x) - D_2 b = 0, \quad \therefore\ D_2 = \frac{1}{b}\left[-R_A a + 1\cdot(a+x)\right] = -\frac{1}{b}\left[\frac{l-x}{l}a-(a+x)\right]$$

$$x = 0 : D_2 = 0, \quad x = l : D_2 = \frac{l+a}{b}$$

これを描くには，図 8.27(c)において，点 B と点 A で-a/b の大きさの点 A' をとり， BA' を O′ まで延長する．そして O′ から点 A に引いた線を延長して，区間 2-4 で修正する

(5)　曲弦トラスの影響線.



D_1-line：　　　　　　　　　　　　　　　図 8.28

$0 \le x \le \lambda$

$$\sum M_{\mathrm{O}} = 0:\ -R_A a + 1\cdot(a+x) + D_1 b = 0,\ \therefore\ D_1 = -\frac{1}{b}\left(1+\frac{a}{l}\right)x,\quad x=0: D_1 = 0,\quad x=l: D_1 = -\frac{l+a}{b}$$

$2\lambda \le x \le l$

$$\sum M_{\mathrm{O}} = 0:\ -R_A a + D_1 b = 0,\quad \therefore\ D_1 = R_A a = \frac{a}{b}\left(\frac{l-x}{l}\right),\quad x=0: D_1 = \frac{a}{b},\ \ x=l: D_1 = 0$$

V_2-line：

$0 \le x \le 2\lambda$

$$\sum M_{\mathrm{O}} = 0:\ -R_A a + 1\cdot(a+x) - V_2 a' = 0,\quad \therefore\ V_2 = \frac{1}{a'}\left(1+\frac{a}{l}\right)x,\quad x=0: V_2 = 0,\quad x=l: V_2 = \frac{l+a}{a'}$$

$3\lambda \le x \le l$

$$\sum M_{\mathrm{O}} = 0:\ -R_A a - V_2 a' = 0,\quad \therefore\ V_2 = -\frac{a}{a'}\left(\frac{l-x}{l}\right),\quad x=0: V_2 = -\frac{a}{a'},\ \ x=l: V_2 = 0$$

V-line を描くには，点 *A* 上で-*a/b* の大きさをとり，点 *B* で 0 の線分を，O′ まで延長する．そして O′ から点 *A* に引いた線を延長して，区間 2-4 で修正する．

(6)　K トラスの影響線.

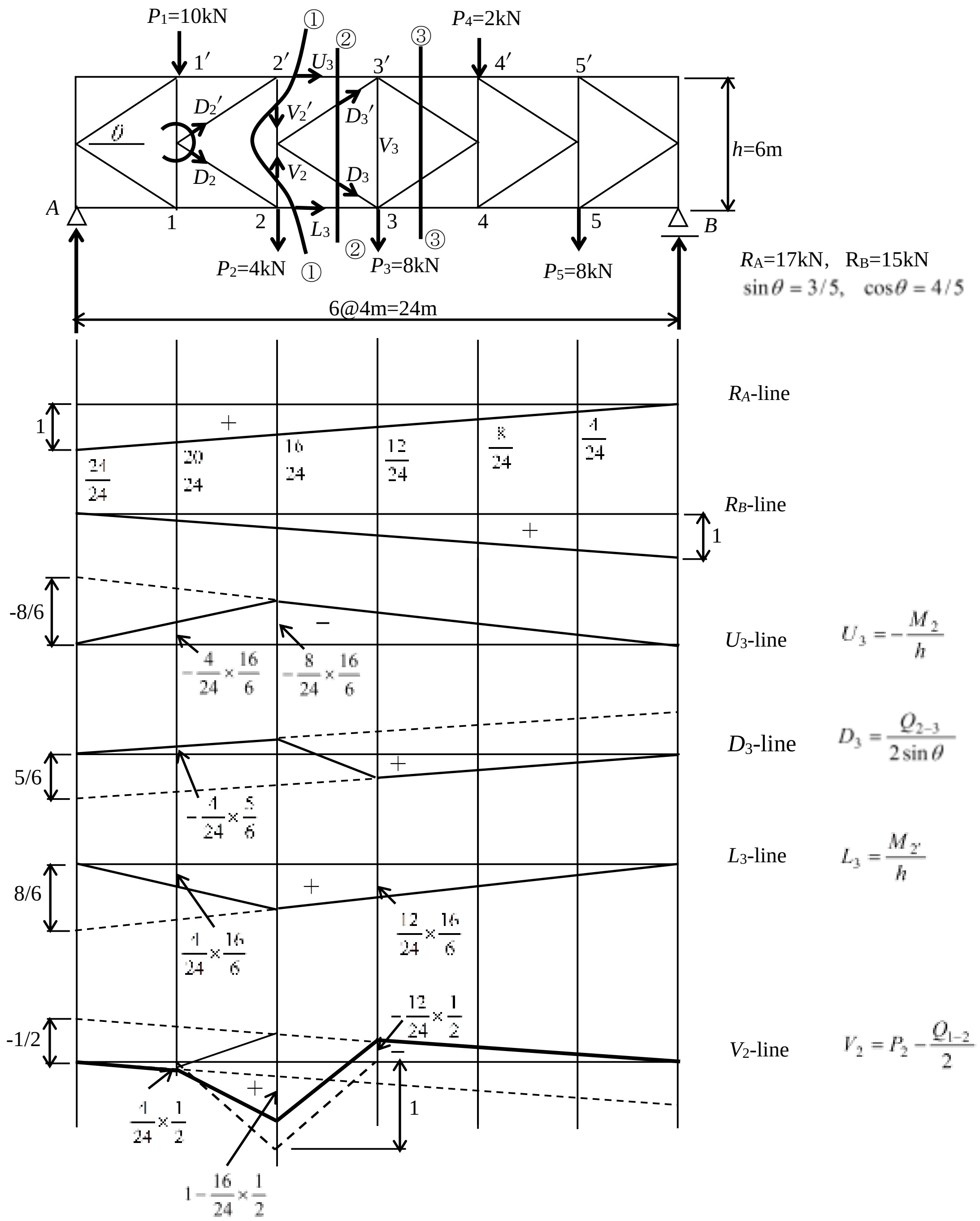


図 8.29