数 学 公 式

三角関数の公式

1.　弧度法

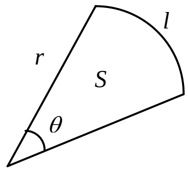
(1)　**ラジアン**　半径 r の円周上にとった長さ r の円弧に対する中心角を 1 ラジアンとする．

1ラジアン $= 180°/\pi = 57°17'45''$　（すなわち，$180°$ は π ラジアン）

(2)　$\alpha°$ は何ラジアン(θ)か ： $\theta = \dfrac{\pi}{180}\alpha$

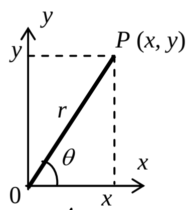
2.　扇形

弧の長さ：$l = r\theta$,　　面積：$S = \dfrac{1}{2}r^2\theta$

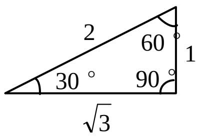


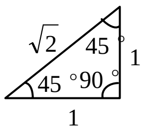
3.　三角関数の定義

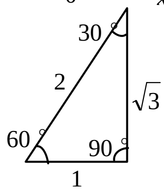
$\sin\theta = \dfrac{y}{r}$,　$\cos\theta = \dfrac{x}{r}$,　$\tan\theta = \dfrac{y}{x}$



角度 θ が $30°, 45°, 60°$ の場合の各辺の長さの関係







4.　三角関数の相互関係

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$,　$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$,　$\mathrm{cosec}\,\theta = \dfrac{1}{\sin\theta}$,　$\sec\theta = \dfrac{1}{\cos\theta}$,　$\cot\theta = \dfrac{1}{\tan\theta}$

$\sin^2\theta + \cos^2\theta = 1$,　$1 + \tan^2\theta = \dfrac{1}{\cos^2\theta}$

5.　角度の関係

$\sin(-\theta) = -\sin\theta$,　$\cos(-\theta) = \cos\theta$,　$\tan(-\theta) = -\tan\theta$

$\sin\left(\dfrac{\pi}{2} \pm \theta\right) = \cos\theta$,　$\cos\left(\dfrac{\pi}{2} \pm \theta\right) = \mp\sin\theta$,　$\tan\left(\dfrac{\pi}{2} \pm \theta\right) = \mp\dfrac{1}{\tan\theta}$

$\sin(\pi \pm \theta) = \mp\sin\theta$,　$\cos(\pi \pm \theta) = -\cos\theta$,　$\tan(\pi \pm \theta) = \pm\tan\theta$

$\sin(2n\pi + \theta) = \sin\theta$,　$\cos(2n\pi + \theta) = \cos\theta$,　$\tan(n\pi + \theta) = \tan\theta$　（n は任意の整数）

6.　加法定理

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

7.　倍角公式

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha \implies \cos^2\alpha = \frac{1+\cos 2\alpha}{2},\ \sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

8.　半角公式

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2},\quad \cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2},\quad \tan^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{1+\cos\alpha}$$

9.　和→積

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2},\quad \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2},\quad \cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

10.　積→和

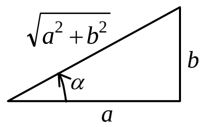
$$\sin\alpha\cos\beta = \frac{1}{2}\{\sin(\alpha+\beta)+\sin(\alpha-\beta)\},\quad \cos\alpha\sin\beta = \frac{1}{2}\{\sin(\alpha+\beta)-\sin(\alpha-\beta)\}$$

$$\cos\alpha\cos\beta = \frac{1}{2}\{\cos(\alpha+\beta)+\cos(\alpha-\beta)\},\quad \sin\alpha\sin\beta = -\frac{1}{2}\{\cos(\alpha+\beta)-\cos(\alpha-\beta)\}$$

11.　三角関数の合成

$$a\sin\theta + b\cos\theta = \sqrt{a^2+b^2}\,\sin(\theta+\alpha)$$

ここに，　$\sin\alpha = \dfrac{b}{\sqrt{a^2+b^2}}$，　$\cos\alpha = \dfrac{a}{\sqrt{a^2+b^2}}$



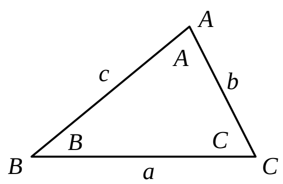
12.　三角形

S=面積，r=内接円半径，R=外接円半径

(1)　$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$　（正弦の法則）

(2)　$a = b\cos C + c\cos B$　（第一余弦の法則）

(3)　$a^2 = b^2 + c^2 - 2bc\cos A$　（第二余弦の法則）



(4)　$\mathrm{S} = \dfrac{1}{2}ab\sin C = \dfrac{a^2\sin B\sin C}{2\sin A} = \sqrt{s(s-a)(s-b)(s-c)}$

$= rs = \dfrac{abc}{4R} = 2R^2\sin A\sin B\sin C = s^2\tan\dfrac{A}{2}\tan\dfrac{B}{2}\tan\dfrac{C}{2}$，　ただし，$s = (a+b+c)/2$

13.　三角関数と指数関数

$\cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2}$，　$\sin\theta = \dfrac{e^{i\theta}-e^{-i\theta}}{2i}$　　（オイラーの公式）

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$　(n:実数)　　（ド・モアブルの公式）

$\cos\theta + i\sin\theta = \mathrm{e}^{i\theta}$，　$\cos\theta - i\sin\theta = \mathrm{e}^{-i\theta}$

14. 双曲線関数と逆双曲線関数

$$\sinh x = \frac{e^x - e^{-x}}{2} = -i \sin ix, \quad \mathrm{cosech} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} = \cos ix, \quad \mathrm{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x - e^{-x}} = -i \tan ix, \quad \coth x = \frac{1}{\tanh x}$$

$$\cosh^2 x = 1 + \sinh^2 x, \quad \mathrm{sec\,h}^2 x = 1 - \tanh^2 x, \quad \mathrm{cosech}^2 x = \coth^2 x - 1$$

$$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y, \quad \cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$$

$$\sinh 2x = 2 \sinh x \cosh x, \quad \cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\sinh^2 x = \frac{\cosh 2x - 1}{2}, \quad \cosh^2 x = \frac{\cosh 2x + 1}{2}$$

$$\sinh^{-1} x = \cosh^{-1} \sqrt{x^2 + 1} = \log(x + \sqrt{x^2 + 1}), \quad \mathrm{cosech}^{-1} x = \sinh^{-1} \frac{1}{x}$$

$$\cosh^{-1} x = \sinh^{-1} \sqrt{x^2 - 1} = \log(x + \sqrt{x^2 - 1}) \ (x > 1), \quad \mathrm{sech}^{-1} x = \cosh^{-1} \frac{1}{x} \ (0 < x < 1)$$

$$\tanh^{-1} x = \frac{1}{2} \log \frac{1+x}{1-x} \ (|x| < 1), \quad \coth^{-1} x = \tanh^{-1} \frac{1}{x} \ (|x| > 1)$$