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等方性･異方性材料における基本式

応力解析を行う場合，複素数を使うと計算が極めて簡単になる場合が多い．そのとき使うと便利な展開式を提示しておく．本計算は，松江高専 25 期卒業生石田知久君（現栗田工業）が計算してくれたものを編集整理したものである．

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1.　等方性材料における基礎方程式

1.1　複素応力関数

等方性材料における合力，変異および応力は複素応力関数$\phi_i(z), \psi_i(z)$によって次式で表される．

$$\left.\begin{aligned} &-P_{yi}+iP_{xi}=\phi_i(z)+z\overline{\phi_i'(z)}+\overline{\psi_i(z)} \\ &2(u_{xi}+iu_{yi})=\frac{1}{G_i}\left\{\kappa_i\phi_i(z)-Z\overline{\phi_i'(z)}-\overline{\psi_i(z)}\right\} \\ &\sigma_{yi}+\sigma_{xi}=\mathrm{Re}\left\{4\phi_i'(z)\right\} \\ &\sigma_{yi}-\sigma_{xi}+2i\tau_{xyi}=2\left\{\bar{z}\phi_i''(z)+\psi_i'(z)\right\} \end{aligned}\right\} \tag{1}$$

ここに

$$z=x+iy=re^{i\theta}, \qquad \bar{z}=x-iy=re^{-i\theta}, \qquad (e^{i\theta}=\cos\theta+i\sin\theta) \tag{2}$$

$$\kappa_i=(3-\nu_i)/(1+\nu_i)\ \text{（平面応力）},\quad \kappa_i=3-4\nu_i\ \text{（平面ひずみ）} \tag{3}$$

ここで，複素応力関数を次式で定義する．

$$\phi_i(z)=a_{1i}z^{\lambda}+a_{2i}z^{\bar{\lambda}}, \qquad \psi_i(z)=b_{1i}z^{\lambda}+b_{2i}z^{\bar{\lambda}}\ {}^{1)} \tag{4}$$

ここに指数λ，係数$a_{1i}, a_{2i}, b_{1i}, b_{2i}$は複素数である．

注 1)　式(4)の第 2 式は　$\psi_i(z)=b_{1i}z^{\lambda+1}/(\lambda+1)+b_{2i}z^{\bar{\lambda}+1}/(\bar{\lambda}+1)$　としても同じである．

$$\left.\begin{aligned} &z=e^{i\theta}, \qquad \bar{z}=e^{-i\theta} \\ &-\mu_1=\mu_1e^{i\pi}, \qquad -\bar{\mu}_1=\bar{\mu}_1e^{-i\pi} \\ &e^{i\theta}=\cos\theta+i\sin\theta, \qquad e^{i\cdot 2\pi}=\cos 2\pi+i\sin 2\pi=1 \\ &e^{i(\lambda-1)\cdot 2\pi}=e^{i\lambda\cdot 2\pi}e^{-i\cdot 2\pi}=e^{2i\lambda\pi}, \qquad e^{-i(\lambda-1)\cdot 2\pi}=e^{-2i\lambda\pi} \\ &e^{i(\lambda+1)\cdot 2\pi}=e^{i\lambda\cdot 2\pi}e^{-i\cdot 2\pi}=e^{2i\lambda\pi}, \qquad e^{-i(\lambda+1)\cdot 2\pi}=e^{-2i\lambda\pi} \end{aligned}\right\} \tag{5}$$

あとの便利のため次の計算をしておく.

$$
\left.
\begin{aligned}
\phi_i(z) &= a_{1i} z^{\lambda} + a_{2i} z^{\bar{\lambda}} \\
&= a_{1i} r^{\lambda} e^{i\lambda\theta} + a_{2i} r^{\bar{\lambda}} e^{i\bar{\lambda}\theta} \\
\phi_i'(z) &= a_{1i} \lambda z^{\lambda-1} + a_{2i} \bar{\lambda} z^{\bar{\lambda}-1} \\
&= a_{1i} \lambda r^{\lambda-1} e^{i(\lambda-1)\theta} + a_{2i} \bar{\lambda} r^{\bar{\lambda}-1} e^{i(\bar{\lambda}-1)\theta} \\
\phi_i''(z) &= a_{1i} \lambda(\lambda-1) z^{\lambda-2} + a_{2i} \bar{\lambda}(\bar{\lambda}-1) z^{\bar{\lambda}-2} \\
&= a_{1i} \lambda(\lambda-1) r^{\lambda-2} e^{i(\lambda-2)\theta} + a_{2i} \bar{\lambda}(\bar{\lambda}-1) r^{\bar{\lambda}-2} e^{i(\bar{\lambda}-2)\theta} \\
\overline{\phi_i(z)} &= \bar{a}_{1i} \bar{z}^{\bar{\lambda}} + \bar{a}_{2i} \bar{z}^{\lambda} \\
&= \bar{a}_{1i} r^{\bar{\lambda}} e^{-i\bar{\lambda}\theta} + \bar{a}_{2i} r^{\lambda} e^{-i\lambda\theta} \\
\overline{\phi_i'(z)} &= \bar{a}_{1i} \bar{\lambda} \bar{z}^{\bar{\lambda}-1} + \bar{a}_{2i} \lambda \bar{z}^{\lambda-1} \\
&= \bar{a}_{1i} \bar{\lambda} r^{\bar{\lambda}-1} e^{-i(\bar{\lambda}-1)\theta} + \bar{a}_{2i} \lambda r^{\lambda-1} e^{-i(\lambda-1)\theta} \\
\overline{\phi_i''(z)} &= \bar{a}_{1i} \bar{\lambda}(\bar{\lambda}-1) \bar{z}^{\bar{\lambda}-2} + \bar{a}_{2i} \lambda(\lambda-1) \bar{z}^{\lambda-2} \\
&= \bar{a}_{1i} \bar{\lambda}(\bar{\lambda}-1) r^{\bar{\lambda}-2} e^{-i(\bar{\lambda}-2)\theta} + \bar{a}_{2i} \lambda(\lambda-1) r^{\lambda-2} e^{-i(\lambda-2)\theta} \\
\bar{z}\phi_i''(z) &= r e^{-i\theta} \phi_i''(z) \\
&= a_{1i} \lambda(\lambda-1) r^{\lambda-1} e^{i(\lambda-3)\theta} + a_{2i} \bar{\lambda}(\bar{\lambda}-1) r^{\bar{\lambda}-1} e^{i(\bar{\lambda}-3)\theta} \\
z\overline{\phi_i'(z)} &= r e^{i\theta} \overline{\phi_i'(z)} \\
&= \bar{a}_{1i} \bar{\lambda} r^{\bar{\lambda}} e^{-i(\bar{\lambda}-2)\theta} + \bar{a}_{2i} \lambda r^{\lambda} e^{-i(\lambda-2)\theta}
\end{aligned}
\right\}
\tag{6}
$$

$$
\left.
\begin{aligned}
\psi_i(z) &= b_{1i} z^{\lambda} + b_{2i} z^{\bar{\lambda}} \\
&= b_{1i} r^{\lambda} e^{i\lambda\theta} + b_{2i} r^{\bar{\lambda}} e^{i\bar{\lambda}\theta} \\
\psi_i'(z) &= b_{1i} \lambda z^{\lambda-1} + b_{2i} \bar{\lambda} z^{\bar{\lambda}-1} \\
&= b_{1i} \lambda r^{\lambda-1} e^{i(\lambda-1)\theta} + b_{2i} \bar{\lambda} r^{\bar{\lambda}-1} e^{i(\bar{\lambda}-1)\theta} \\
\psi_i''(z) &= b_{1i} \lambda(\lambda-1) z^{\lambda-2} + b_{2i} \bar{\lambda}(\bar{\lambda}-1) z^{\bar{\lambda}-2} \\
&= b_{1i} \lambda(\lambda-1) r^{\lambda-2} e^{i(\lambda-2)\theta} + b_{2i} \bar{\lambda}(\bar{\lambda}-1) r^{\bar{\lambda}-2} e^{i(\bar{\lambda}-2)\theta} \\
\overline{\psi_i(z)} &= \bar{b}_{1i} \bar{z}^{\bar{\lambda}} + \bar{b}_{2i} \bar{z}^{\lambda} \\
&= \bar{b}_{1i} r^{\bar{\lambda}} e^{-i\bar{\lambda}\theta} + \bar{b}_{2i} r^{\lambda} e^{-i\lambda\theta} \\
\overline{\psi_i'(z)} &= \bar{b}_{1i} \bar{\lambda} \bar{z}^{\bar{\lambda}-1} + \bar{b}_{2i} \lambda \bar{z}^{\lambda-1} \\
&= \bar{b}_{1i} \bar{\lambda} r^{\bar{\lambda}-1} e^{-i(\bar{\lambda}-1)\theta} + \bar{b}_{2i} \lambda r^{\lambda-1} e^{-i(\lambda-1)\theta} \\
\overline{\psi_i''(z)} &= \bar{b}_{1i} \bar{\lambda}(\bar{\lambda}-1) \bar{z}^{\bar{\lambda}-2} + \bar{b}_{2i} \lambda(\lambda-1) \bar{z}^{\lambda-2} \\
&= \bar{b}_{1i} \bar{\lambda}(\bar{\lambda}-1) r^{\bar{\lambda}-2} e^{-i(\bar{\lambda}-2)\theta} + \bar{b}_{2i} \lambda(\lambda-1) r^{\lambda-1} e^{-i(\lambda-2)\theta} \\
\bar{z}\psi_i''(z) &= r e^{-i\theta} \psi_i''(z) \\
&= b_{1i} \lambda(\lambda-1) r^{\lambda-1} e^{i(\lambda-3)\theta} + b_{2i} \bar{\lambda}(\bar{\lambda}-1) r^{\bar{\lambda}-1} e^{i(\bar{\lambda}-3)\theta} \\
z\overline{\psi_i'(z)} &= r e^{i\theta} \overline{\psi_i'(z)} \\
&= \bar{b}_{1i} \bar{\lambda} r^{\bar{\lambda}} e^{-i(\bar{\lambda}-2)\theta} + \bar{b}_{2i} \lambda r^{\lambda} e^{-i(\lambda-2)\theta}
\end{aligned}
\right\}
\tag{7}
$$

1.2 $x-y$ 座標系基本式

1) $-P_{yi}+iP_{xi}$

$$=\phi_i(z)+z\overline{\phi_i'(z)}+\overline{\psi_i(z)}$$

$$=a_{1i}r^{\lambda}e^{i\lambda\theta}+a_{2i}r^{\bar\lambda}e^{i\bar\lambda\theta}+\bar a_{1i}\bar\lambda r^{\bar\lambda}e^{-i(\bar\lambda-2)\theta}+\bar a_{2i}\lambda r^{\lambda}e^{-i(\lambda-2)\theta}+\bar b_{1i}r^{\bar\lambda}e^{-i\bar\lambda\theta}+\bar b_{2i}r^{\lambda}e^{-i\lambda\theta} \tag{8}$$

2) $2(u_{xi}+iu_{yi})$

$$=\frac{1}{G_i}\left\{\kappa_i\phi_i(z)-z\overline{\phi_i'(z)}-\overline{\psi_i(z)}\right\}$$

$$=\frac{1}{G_i}\left\{a_{1i}\kappa_i r^{\lambda}e^{i\lambda\theta}+a_{2i}\kappa_i r^{\bar\lambda}e^{i\bar\lambda\theta}-\bar a_{1i}\bar\lambda r^{\bar\lambda}e^{-i(\bar\lambda-2)\theta}-\bar a_{2i}\lambda r^{\lambda}e^{-i(\lambda-2)\theta}-\bar b_{1i}r^{\bar\lambda}e^{-i\bar\lambda\theta}-\bar b_{2i}r^{\lambda}e^{-i\lambda\theta}\right\} \tag{9}$$

3) $\sigma_{yi}+\sigma_{xi}=\mathrm{Re}\{4\phi_i'(z)\}$

$$=\mathrm{Re}\left\{4a_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+4a_{2i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta}\right\} \tag{10}$$

4) $\sigma_{yi}-\sigma_{xi}+2i\tau_{xyi}$

$$=2\left\{\bar z\phi''(z)+\psi_i'(z)\right\}$$

$$=2\left\{a_{1i}\lambda(\lambda-1)r^{\lambda-1}e^{i(\lambda-3)\theta}+a_{2i}\bar\lambda(\bar\lambda-1)r^{\bar\lambda-1}e^{i(\bar\lambda-3)\theta}+b_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+b_{2i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta}\right\} \tag{11}$$

5) {(10)+(11)}/2

$\sigma_{yi}+i\tau_{xyi}$

$$=\mathrm{Re}\left\{2a_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+2a_{2i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta}\right\}$$
$$+a_{1i}\lambda(\lambda-1)r^{\lambda-1}e^{i(\lambda-3)\theta}+a_{2i}\bar\lambda(\bar\lambda-1)r^{\bar\lambda-1}e^{i(\bar\lambda-3)\theta}+b_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+b_{2i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta} \tag{12}$$

6) {(10)-(11)}/2

$\sigma_{xi}-i\tau_{xyi}$

$$=\mathrm{Re}\left\{2a_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+2a_{2i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta}\right\}$$
$$-a_{1i}\lambda(\lambda-1)r^{\lambda-1}e^{i(\lambda-3)\theta}-a_{2i}\bar\lambda(\bar\lambda-1)r^{\bar\lambda-1}e^{i(\bar\lambda-3)\theta}-b_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}-b_{2i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta} \tag{13}$$

1.3 極座標系基本式

$$\left.\begin{array}{lll} z_1=ze^{-i\theta}, & \bar z_1=\bar z e^{i\theta} & \\ \phi_1(z_1)=\phi(z)e^{-i\theta}, & \phi_1'(z_1)=\phi'(z), & \phi_1''(z_1)=\phi''(z)e^{i\theta} \\ \psi_1(z_1)=\psi(z), & \psi_1'(z_1)=\psi'(z)e^{i\theta}, & \psi_1''(z_1)=\psi''(z)e^{2i\theta} \end{array}\right\} \tag{14}$$

1) $-P_{\theta i}+iP_{ri}=\phi_i(z)e^{-i\theta}+z\overline{\phi_i'(z)}e^{-i\theta}+\overline{\psi_i(z)}e^{-i\theta}$

$$=e^{-i\theta}\left\{a_{1i}r^{\lambda}e^{i\lambda\theta}+a_{2i}r^{\bar\lambda}e^{i\bar\lambda\theta}+\bar a_{1i}\bar\lambda r^{\bar\lambda}e^{-i(\bar\lambda-2)\theta}+\bar a_{2i}\lambda r^{\lambda}e^{-i(\lambda-2)\theta}+\bar b_{1i}r^{\bar\lambda}e^{-i\bar\lambda\theta}+\bar b_{2i}r^{\lambda}e^{-i\lambda\theta}\right\}$$

$$=a_{1i}r^{\lambda}e^{i(\lambda-1)\theta}+a_{2i}r^{\bar\lambda}e^{i(\bar\lambda-1)\theta}+\bar a_{1i}\bar\lambda r^{\bar\lambda}e^{-i(\bar\lambda-1)\theta}+\bar a_{2i}\lambda r^{\lambda}e^{-i(\lambda-1)\theta}+\bar b_{1i}r^{\bar\lambda}e^{-i(\bar\lambda+1)\theta}+\bar b_{2i}r^{\lambda}e^{-i(\lambda+1)\theta} \tag{15}$$

2) $$2(u_{ri}+iu_{\theta i})=\frac{e^{-i\theta}}{G_i}\left\{\kappa_i\phi_i(z)-z\overline{\phi_i'(z)}-\overline{\psi_i(z)}\right\}$$
$$=\frac{e^{-i\theta}}{G_i}\left\{a_{1i}\kappa_i r^{\lambda}e^{i\lambda\theta}+a_{2i}\kappa_i r^{\bar\lambda}e^{i\bar\lambda\theta}-\bar a_{1i}\bar\lambda r^{\bar\lambda}e^{-i(\bar\lambda-2)\theta}-\bar a_{2i}\lambda r^{\lambda}e^{-i(\lambda-2)\theta}-\bar b_{1i}r^{\bar\lambda}e^{-i\bar\lambda\theta}-\bar b_{2i}r^{\lambda}e^{-i\lambda\theta}\right\}$$
$$=\frac{1}{G_i}\left\{a_{1i}\kappa_i r^{\lambda}e^{i(\lambda-1)\theta}+a_{2i}\kappa_i r^{\bar\lambda}e^{i(\bar\lambda-1)\theta}-\bar a_{1i}\bar\lambda r^{\bar\lambda}e^{-i(\bar\lambda-1)\theta}-\bar a_{2i}\lambda r^{\lambda}e^{-i(\lambda-1)\theta}-\bar b_{1i}r^{\bar\lambda}e^{-i(\bar\lambda+1)\theta}-\bar b_{2i}r^{\lambda}e^{-i(\lambda+1)\theta}\right\} \tag{16}$$

3) $$\sigma_{\theta i}+\sigma_{ri}=\mathrm{Re}\left\{4\phi_i'(z)\right\}$$
$$=\mathrm{Re}\left\{4a_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+4a_{2i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta}\right\} \tag{17}$$

4) $$\sigma_{\theta i}-\sigma_{ri}+2i\tau_{r\theta i}=2\left\{\bar z\phi_i''(z)+\psi_i'(z)\right\}e^{i\theta}$$
$$=2\left\{\bar z\phi_i''(z)e^{i\theta}e^{i\theta}+\psi_i'(z)e^{i\theta}e^{i\theta}\right\}$$
$$=2e^{2i\theta}\left\{\bar z\phi_i''(z)+\psi_i'(z)\right\}$$
$$=2e^{2i\theta}\left\{a_{1i}\lambda(\lambda-1)r^{\lambda-1}e^{i(\lambda-3)\theta}+a_{2i}\bar\lambda(\bar\lambda-1)r^{\bar\lambda-1}e^{i(\bar\lambda-3)\theta}+b_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+b_{2i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta}\right\}$$
$$=2\left\{a_{1i}\lambda(\lambda-1)r^{\lambda-1}e^{i(\lambda-1)\theta}+a_{2i}\bar\lambda(\bar\lambda-1)r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta}+b_{1i}\lambda r^{\lambda-1}e^{i(\lambda+1)\theta}+b_{2i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda+1)\theta}\right\} \tag{18}$$

5) {(17)+(18)}/2

$$\sigma_{\theta i}+i\tau_{r\theta i}=\mathrm{Re}\left\{2a_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+2a_{2i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta}\right\}^{*}$$
＊ 実数だけのため $z=\bar z$ の形
$$+a_{1i}\lambda(\lambda-1)r^{\lambda-1}e^{i(\lambda-1)\theta}+a_{2i}\bar\lambda(\bar\lambda-1)r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta}+b_{1i}\lambda r^{\lambda-1}e^{i(\lambda+1)\theta}+b_{2i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda+1)\theta}$$
$$=\bar a_{1i}\bar\lambda r^{\bar\lambda-1}e^{-i(\bar\lambda-1)\theta}+\bar a_{2i}\lambda r^{\lambda-1}e^{-i(\lambda-1)\theta}+a_{1i}\lambda^2 r^{\lambda-1}e^{i(\lambda-1)\theta}$$
$$+a_{2i}\bar\lambda^2 r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta}+b_{1i}\lambda r^{\lambda-1}e^{i(\lambda+1)\theta}+b_{2i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda+1)\theta}$$
$$=a_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+\bar a_{2i}\lambda r^{\lambda-1}e^{-i(\lambda-1)\theta}+a_{1i}\lambda^2 r^{\lambda-1}e^{i(\lambda-1)\theta}$$
$$+\bar a_{2i}\lambda^2 r^{\lambda-1}e^{-i(\bar\lambda-1)\theta}+b_{1i}\lambda r^{\lambda-1}e^{i(\lambda+1)\theta}+\bar b_{2i}\lambda r^{\lambda-1}e^{-i(\lambda+1)\theta} \tag{19}$$

6) {(17)-(18)}/2

$$\sigma_{ri}-i\tau_{r\theta i}=\mathrm{Re}\left\{2a_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+2a_{2i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta}\right\}^{*}$$
＊ 実数だけのため $z=\bar z$ の形
$$-a_{1i}\lambda(\lambda-1)r^{\lambda-1}e^{i(\lambda-1)\theta}-a_{2i}\bar\lambda(\bar\lambda-1)r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta}-b_{1i}\lambda r^{\lambda-1}e^{i(\lambda+1)\theta}-b_{2i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda+1)\theta}$$
$$=2\bar a_{1i}\bar\lambda r^{\bar\lambda-1}e^{-i(\bar\lambda-1)\theta}+2\bar a_{2i}\lambda r^{\lambda-1}e^{-i(\lambda-1)\theta}$$
$$-a_{1i}\lambda(\lambda-1)r^{\lambda-1}e^{i(\lambda-1)\theta}-a_{2i}\bar\lambda(\bar\lambda-1)r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta}-b_{1i}\lambda r^{\lambda-1}e^{i(\lambda+1)\theta}-b_{2i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda+1)\theta}$$
$$=3a_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+3\bar a_{2i}\lambda r^{\lambda-1}e^{-i(\lambda-1)\theta}-a_{1i}\lambda^2 r^{\lambda-1}e^{i(\lambda-1)\theta}$$
$$-\bar a_{2i}\lambda^2 r^{\lambda-1}e^{-i(\lambda-1)\theta}-b_{1i}\lambda r^{\lambda-1}e^{i(\lambda+1)\theta}-\bar b_{2i}\lambda r^{\lambda-1}e^{-i(\lambda+1)\theta}$$
$$=3\bar a_{1i}\bar\lambda r^{\bar\lambda-1}e^{i(\bar\lambda-1)\theta}+3\bar a_{2i}\lambda r^{\lambda-1}e^{-i(\lambda-1)\theta}-a_{1i}\lambda^2 r^{\lambda-1}e^{i(\lambda-1)\theta}$$
$$-a_{2i}\bar\lambda^2 r^{\bar\lambda-1}e^{-i(\bar\lambda-1)\theta}-b_{1i}\lambda r^{\lambda-1}e^{i(\lambda+1)\theta}-b_{2i}\bar\lambda r^{\bar\lambda-1}e^{-i(\bar\lambda+1)\theta} \tag{20}$$

1.4 特別な場合（$x-y$ 座標系）

1） 式(8)の実部のみ（$z=\bar z$）をとる．

$$-P_{yi}=a_{1i}r^{\lambda}e^{i\lambda\theta}+a_{2i}r^{\bar\lambda}e^{i\bar\lambda\theta}+\bar a_{1i}\bar\lambda r^{\bar\lambda}e^{-i(\bar\lambda-2)\theta}+\bar a_{2i}\lambda r^{\lambda}e^{-i(\lambda-2)\theta}+\bar b_{1i}r^{\bar\lambda}e^{-i\bar\lambda\theta}+\bar b_{2i}r^{\lambda}e^{-i\lambda\theta}$$
$$=a_{1i}r^{\lambda}e^{i\lambda\theta}+\bar a_{2i}r^{\lambda}e^{-i\lambda\theta}+a_{1i}\lambda r^{\lambda}e^{i(\lambda-2)\theta}+\bar a_{2i}\lambda r^{\lambda}e^{-i(\lambda-2)\theta}+b_{1i}r^{\lambda}e^{i\lambda\theta}+\bar b_{2i}r^{\lambda}e^{-i\lambda\theta}$$
$$=a_{1i}r^{\lambda}(1+\lambda e^{-2i\theta})e^{i\lambda\theta}+\bar a_{2i}r^{\lambda}(1+\lambda e^{2i\theta})e^{-i\lambda\theta}+b_{1i}r^{\lambda}e^{i\lambda\theta}+\bar b_{2i}r^{\lambda}e^{-i\lambda\theta} \tag{21}$$

2) 式(8)の虚部のみ（$z=-\bar{z}$）をとる．

$$iP_{xi}=a_{1i}r^{\lambda}e^{i\lambda\theta}+a_{2i}r^{\bar{\lambda}}e^{i\bar{\lambda}\theta}+\bar{a}_{1i}\bar{\lambda}r^{\bar{\lambda}}e^{-i(\bar{\lambda}-2)\theta}+\bar{a}_{2i}\lambda r^{\lambda}e^{-i(\lambda-2)\theta}+\bar{b}_{1i}r^{\bar{\lambda}}e^{-i\bar{\lambda}\theta}+\bar{b}_{2i}r^{\lambda}e^{-i\lambda\theta}$$
$$=a_{1i}r^{\lambda}e^{i\lambda\theta}-\bar{a}_{2i}r^{\lambda}e^{-i\lambda\theta}-a_{1i}\lambda r^{\lambda}e^{i(\lambda-2)\theta}+\bar{a}_{2i}\lambda r^{\lambda}e^{-i(\lambda-2)\theta}-b_{1i}r^{\lambda}e^{i\lambda\theta}+\bar{b}_{2i}r^{\lambda}e^{-i\lambda\theta}$$
$$=a_{1i}r^{\lambda}(1-\lambda e^{-2i\theta})e^{i\lambda\theta}-\bar{a}_{2i}r^{\lambda}(1-\lambda e^{2i\theta})e^{-i\lambda\theta}-b_{1i}r^{\lambda}e^{i\lambda\theta}+\bar{b}_{2i}r^{\lambda}e^{-i\lambda\theta} \tag{22}$$

3) 式(9)の実部のみ（$z=\bar{z}$）をとる．

$$2u_{xi}=\frac{1}{G_i}\left\{a_{1i}\kappa_i r^{\lambda}e^{i\lambda\theta}+a_{2i}\kappa_i r^{\bar{\lambda}}e^{i\bar{\lambda}\theta}-\bar{a}_{1i}\bar{\lambda}r^{\bar{\lambda}}e^{-i(\bar{\lambda}-2)\theta}-\bar{a}_{2i}\lambda r^{\lambda}e^{-i(\lambda-2)\theta}-\bar{b}_{1i}r^{\bar{\lambda}}e^{-i\bar{\lambda}\theta}-\bar{b}_{2i}r^{\lambda}e^{-i\lambda\theta}\right\}$$
$$=\frac{1}{G_i}\left\{a_{1i}\kappa_i r^{\lambda}e^{i\lambda\theta}+\bar{a}_{2i}\kappa_i r^{\lambda}e^{-i\lambda\theta}-a_{1i}\lambda r^{\lambda}e^{i(\lambda-2)\theta}-\bar{a}_{2i}\lambda r^{\lambda}e^{-i(\lambda-2)\theta}-b_{1i}r^{\lambda}e^{i\lambda\theta}-\bar{b}_{2i}r^{\lambda}e^{-i\lambda\theta}\right\}$$
$$=\frac{1}{G_i}\left\{a_{1i}r^{\lambda}(\kappa_i-\lambda e^{-2i\theta})e^{i\lambda\theta}+\bar{a}_{2i}r^{\lambda}(\kappa_i-\lambda e^{2i\theta})e^{-i\lambda\theta}-b_{1i}r^{\lambda}e^{i\lambda\theta}-\bar{b}_{2i}r^{\lambda}e^{-i\lambda\theta}\right\} \tag{23}$$

4) 式(9)の虚部のみ（$z=-\bar{z}$）をとる．

$$2iu_{yi}=\frac{1}{G_i}\left\{a_{1i}\kappa_i r^{\lambda}e^{i\lambda\theta}+a_{2i}\kappa_i r^{\bar{\lambda}}e^{i\bar{\lambda}\theta}-\bar{a}_{1i}\bar{\lambda}r^{\bar{\lambda}}e^{-i(\bar{\lambda}-2)\theta}-\bar{a}_{2i}\lambda r^{\lambda}e^{-i(\lambda-2)\theta}-\bar{b}_{1i}r^{\bar{\lambda}}e^{-i\bar{\lambda}\theta}-\bar{b}_{2i}r^{\lambda}e^{-i\lambda\theta}\right\}$$
$$=\frac{1}{G_i}\left\{a_{1i}\kappa_i r^{\lambda}e^{i\lambda\theta}-\bar{a}_{2i}\kappa_i r^{\lambda}e^{-i\lambda\theta}+a_{1i}\lambda r^{\lambda}e^{i(\lambda-2)\theta}-\bar{a}_{2i}\lambda r^{\lambda}e^{-i(\lambda-2)\theta}+b_{1i}r^{\lambda}e^{i\lambda\theta}-\bar{b}_{2i}r^{\lambda}e^{-i\lambda\theta}\right\}$$
$$=\frac{1}{G_i}\left\{a_{1i}r^{\lambda}(\kappa_i+\lambda e^{-2i\theta})e^{i\lambda\theta}-\bar{a}_{2i}r^{\lambda}(\kappa_i+\lambda e^{2i\theta})e^{-i\lambda\theta}+b_{1i}r^{\lambda}e^{i\lambda\theta}-\bar{b}_{2i}r^{\lambda}e^{-i\lambda\theta}\right\} \tag{24}$$

5) 式(12)の実部のみ（$z=\bar{z}$）をとる．

$$\sigma_{yi}=2a_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+2a_{2i}\bar{\lambda}r^{\bar{\lambda}-1}e^{i(\bar{\lambda}-1)\theta}+a_{1i}\lambda(\lambda-1)r^{\lambda-1}e^{i(\lambda-3)\theta}$$
$$+a_{2i}\bar{\lambda}(\bar{\lambda}-1)r^{\bar{\lambda}-1}e^{i(\bar{\lambda}-3)\theta}+b_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+b_{2i}\bar{\lambda}r^{\bar{\lambda}-1}e^{i(\bar{\lambda}-1)\theta}$$
$$=2a_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+2\bar{a}_{2i}\lambda r^{\lambda-1}e^{-i(\lambda-1)\theta}+a_{1i}\lambda(\lambda-1)r^{\lambda-1}e^{i(\lambda-3)\theta}$$
$$+\bar{a}_{2i}\lambda(\lambda-1)r^{\lambda-1}e^{-i(\lambda-3)\theta}+b_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+\bar{b}_{2i}\lambda r^{\lambda-1}e^{-i(\lambda-1)\theta}$$
$$=a_{1i}\lambda r^{\lambda-1}\left\{2+(\lambda-1)e^{-2i\theta}\right\}e^{i(\lambda-1)\theta}+\bar{a}_{2i}\lambda r^{\lambda-1}\left\{2+(\lambda-1)e^{2i\theta}\right\}e^{-i(\lambda-1)\theta}+b_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+\bar{b}_{2i}\lambda r^{\lambda-1}e^{-i(\lambda-1)\theta} \tag{25}$$

6) 式(12)の虚部のみ（$z=-\bar{z}$）をとる．

$$i\tau_{xyi}=a_{1i}\lambda(\lambda-1)r^{\lambda-1}e^{i(\lambda-3)\theta}+a_{2i}\bar{\lambda}(\bar{\lambda}-1)r^{\bar{\lambda}-1}e^{i(\bar{\lambda}-3)\theta}+b_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+b_{2i}\bar{\lambda}r^{\bar{\lambda}-1}e^{i(\bar{\lambda}-1)\theta}$$
$$=a_{1i}\lambda(\lambda-1)r^{\lambda-1}e^{i(\lambda-3)\theta}-\bar{a}_{2i}\lambda(\lambda-1)r^{\lambda-1}e^{-i(\lambda-3)\theta}+b_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}-\bar{b}_{2i}\lambda r^{\lambda-1}e^{-i(\lambda-1)\theta}$$
$$=a_{1i}\lambda r^{\lambda-1}\left\{(\lambda-1)e^{-2i\theta}\right\}e^{i(\lambda-1)\theta}-\bar{a}_{2i}\lambda r^{\lambda-1}\left\{(\lambda-1)e^{2i\theta}\right\}e^{-i(\lambda-1)\theta}+b_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}-\bar{b}_{2i}\lambda r^{\lambda-1}e^{-i(\lambda-1)\theta} \tag{26}$$

7) 式(13)の実部のみ（$z=\bar{z}$）をとる．

$$\sigma_{xi}=2a_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+2a_{2i}\bar{\lambda}r^{\bar{\lambda}-1}e^{i(\bar{\lambda}-1)\theta}-a_{1i}\lambda(\lambda-1)r^{\lambda-1}e^{i(\lambda-3)\theta}$$
$$-a_{2i}\bar{\lambda}(\bar{\lambda}-1)r^{\bar{\lambda}-1}e^{i(\bar{\lambda}-3)\theta}-b_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}-b_{2i}\bar{\lambda}r^{\bar{\lambda}-1}e^{i(\bar{\lambda}-1)\theta}$$
$$=2a_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}+2\bar{a}_{2i}\lambda r^{\lambda-1}e^{-i(\lambda-1)\theta}-a_{1i}\lambda(\lambda-1)r^{\lambda-1}e^{i(\lambda-3)\theta}$$
$$-\bar{a}_{2i}\lambda(\lambda-1)r^{\lambda-1}e^{-i(\lambda-3)\theta}-b_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}-\bar{b}_{2i}\lambda r^{\lambda-1}e^{-i(\lambda-1)\theta}$$
$$=a_{1i}\lambda r^{\lambda-1}\left\{2-(\lambda-1)e^{-2i\theta}\right\}e^{i(\lambda-1)\theta}+\bar{a}_{2i}\lambda r^{\lambda-1}\left\{2-(\lambda-1)e^{2i\theta}\right\}e^{-i(\lambda-1)\theta}$$
$$-b_{1i}\lambda r^{\lambda-1}e^{i(\lambda-1)\theta}-\bar{b}_{2i}\lambda r^{\lambda-1}e^{-i(\lambda-1)\theta} \tag{27}$$

2. 異方性材料の基礎方程式の整理

2.1 基本式

異方性材における $x-y$ 面上の応力成分とひずみ成分の関係式

$$\left.\begin{aligned}\varepsilon_{xi}&=\frac{\sigma_{xi}}{E_{xi}}-\frac{\nu_{yxi}}{E_{yi}}\sigma_{yi}\\ \varepsilon_{yi}&=\frac{\sigma_{yi}}{E_{yi}}-\frac{\nu_{xyi}}{E_{xi}}\sigma_{xi}\\ \gamma_{xyi}&=\frac{1}{G_{xyi}}\tau_{xyi}\\ \frac{\nu_{xyi}}{E_{xi}}&=\frac{\nu_{yxi}}{E_{yi}}\end{aligned}\right\}\tag{28}$$

直交異方性材料における合力，変位および応力は Lekhnitskii によれば，複素応力関数 $\phi_i(z),\psi_i(z)$ によって次式で表される．

$$\left.\begin{aligned}P_{xi}&=2\,\mathrm{Re}\{\mu_{1i}\phi_{ai}(z_{1i})+\mu_{2i}\phi_{bi}(z_{2i})\}\\ P_{yi}&=-2\,\mathrm{Re}\{\phi_{ai}(z_{1i})+\phi_{bi}(z_{2i})\}\\ u_{xi}&=2\,\mathrm{Re}\{p_{1i}\phi_{ai}(z_{1i})+p_{2i}\phi_{bi}(z_{2i})\}\\ u_{yi}&=2\,\mathrm{Re}\{q_{1i}\phi_{ai}(z_{1i})+q_{2i}\phi_{bi}(z_{2i})\}\\ \sigma_{xi}&=2\,\mathrm{Re}\{\mu_{1i}^2\phi'_{ai}(z_{1i})+\mu_{2i}^2\phi'_{bi}(z_{2i})\}\\ \sigma_{yi}&=2\,\mathrm{Re}\{\phi'_{ai}(z_{1i})+\phi'_{bi}(z_{2i})\}\end{aligned}\right\}\tag{29}$$

複素応力関数は次のべき表示で表す．

$$\left.\begin{aligned}\phi_{ai}(z_{1i})&=a_{1i}z_{1i}{}^{\lambda}+a_{2i}z_{1i}{}^{\bar{\lambda}}\\ \phi_{bi}(z_{2i})&=b_{1i}z_{2i}{}^{\lambda}+b_{2i}z_{2i}{}^{\bar{\lambda}}\end{aligned}\right\}\tag{30}$$

ここに，

$$\left.\begin{aligned}z_{1i}&=x+\mu_{1i}y=re^{\mu_{1i}\theta},\qquad \mu_{1i}=\alpha_x+i\beta_y=l_{1i}e^{i\arg(\mu_{1i})}\\ z_{2i}&=x+\mu_{2i}y=re^{\mu_{2i}\theta},\qquad \mu_{2i}=\alpha_x+i\beta_y=l_{2i}e^{i\arg(\mu_{2i})}\end{aligned}\right\}\tag{31}$$

とし， $\mu_{ji}\ (j=1,2)$ は，

$$\frac{1}{E_{xi}}\mu_{ji}{}^4+\left(-\frac{2\nu_{xyi}}{E_{xi}}+\frac{1}{G_{xyi}}\right)\mu_{ji}{}^2+\frac{1}{E_{yi}}=0\tag{32}$$

の四つの根の中の虚数部が正となる二つの根である．また，

$$\left.\begin{aligned}p_{1i}&=\frac{1}{E_{xi}}\mu_{1i}{}^2-\frac{\nu_{xyi}}{E_{xi}}\\ p_{2i}&=\frac{1}{E_{xi}}\mu_{2i}{}^2-\frac{\nu_{xyi}}{E_{xi}}\\ q_{1i}&=-\frac{\nu_{xyi}}{E_{xi}}\mu_{1i}+\frac{1}{E_{yi}\mu_{1i}}\\ q_{2i}&=-\frac{\nu_{xyi}}{E_{xi}}\mu_{2i}+\frac{1}{E_{yi}\mu_{2i}}\end{aligned}\right\}\tag{33}$$

である．

$$
\begin{aligned}
z_{1i} &= x + \mu_{1i} y \\
&= re^{\mu_{1i}\theta} \\
&= r(\cos\theta + \mu_{1i}\sin\theta) = r(\cos\theta + l_{1i}e^{i\arg(\mu_{1i})}\cdot\sin\theta)
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
z_{2i} &= x + \mu_{2i} y \\
&= re^{\mu_{2i}\theta} \\
&= r(\cos\theta + \mu_{2i}\sin\theta) = r(\cos\theta + l_{2i}e^{i\arg(\mu_{2i})}\cdot\sin\theta)
\end{aligned} \tag{35}
$$

$$
\left.
\begin{aligned}
\phi_{ai}(z_{1i}) &= a_{1i}{z_{1i}}^{\lambda} + a_{2i}{z_{1i}}^{\bar{\lambda}} \\
&= a_{1i}r^{\lambda}(\cos\theta + \mu_{1i}\sin\theta)^{\lambda} + a_{2i}r^{\bar{\lambda}}(\cos\theta + \mu_{1i}\sin\theta)^{\bar{\lambda}} \\
\phi_{bi}(z_{2i}) &= b_{1i}{z_{2i}}^{\lambda} + b_{2i}{z_{2i}}^{\bar{\lambda}} \\
&= b_{1i}r^{\lambda}(\cos\theta + \mu_{2i}\sin\theta)^{\lambda} + b_{2i}r^{\bar{\lambda}}(\cos\theta + \mu_{2i}\sin\theta)^{\bar{\lambda}} \\
\phi'_{ai}(z_{1i}) &= a_{1i}\lambda {z_{1i}}^{\lambda-1} + a_{2i}\bar{\lambda}{z_{1i}}^{\bar{\lambda}-1} \\
&= a_{1i}\lambda r^{\lambda-1}(\cos\theta + \mu_{1i}\sin\theta)^{\lambda-1} + a_{2i}\bar{\lambda}r^{\bar{\lambda}-1}(\cos\theta + \mu_{1i}\sin\theta)^{\bar{\lambda}-1} \\
\phi'_{bi}(z_{2i}) &= b_{1i}\lambda {z_{2i}}^{\lambda-1} + b_{2i}\bar{\lambda}{z_{2i}}^{\bar{\lambda}-1} \\
&= b_{1i}\lambda r^{\lambda-1}(\cos\theta + \mu_{2i}\sin\theta)^{\lambda-1} + b_{2i}\bar{\lambda}r^{\bar{\lambda}-1}(\cos\theta + \mu_{2i}\sin\theta)^{\bar{\lambda}-1}
\end{aligned}
\right\} \tag{36}
$$

注）式(36)第 1 式は式(34)によって次のように変形できる.

$$
\begin{aligned}
\phi_{ai}(z_{1i}) &= a_{1i}{z_{1i}}^{\lambda} + a_{2i}{z_{1i}}^{\bar{\lambda}} \\
&= a_{1i}r^{\lambda}(\cos\theta + \mu_{1i}\sin\theta)^{\lambda} + a_{2i}r^{\bar{\lambda}}(\cos\theta + \mu_{1i}\sin\theta)^{\bar{\lambda}} \\
&= a_{1i}r^{\lambda}(\cos\theta + l_{1i}e^{i\arg(\mu_{1i})}\sin\theta)^{\lambda} + a_{2i}r^{\bar{\lambda}}(\cos\theta + l_{1i}e^{i\arg(\mu_{1i})}\sin\theta)^{\bar{\lambda}}
\end{aligned}
$$

他も同様である.

注） $z_{1i} = x + \mu_{1i} = \dfrac{1}{2}z(1 - i\mu_{1i}) + \dfrac{1}{2}(1 + i\mu_{1i})$

$$\arg(1) = \tan^{-1}1 = 45°, \quad \arg\left(\frac{1}{\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$

$$\theta = \arg(\mu) = \tan^{-1}\frac{\mathrm{Im}(\mu)}{\mathrm{Re}(\mu)}$$

$$e^{i\cdot 90°} = \cos 90° + i\sin 90° = 1$$

$$l = |\mu| = \sqrt{(\mathrm{Re}(\mu))^2 + (\mathrm{Im}(\mu))^2}$$

$$\mu_{1i} = l_{1i}e^{i\arg(\mu_{1i})}, \quad \bar{\mu}_{1i} = l_{1i}e^{-i\arg(\mu_{1i})}$$

2.2 $x-y$ 座標系基本式

1) $$P_{xi}=2\,\mathrm{Re}\left\{\mu_{1i}\phi_{ai}(z_{1i})+\mu_{2i}\phi_{bi}(z_{2i})\right\}$$
$$=2\,\mathrm{Re}\Big\{a_{1i}\mu_{1i}r^{\lambda}(\cos\theta+\mu_{1i}\sin\theta)^{\lambda}+a_{2i}\mu_{1i}r^{\bar{\lambda}}(\cos\theta+\mu_{1i}\sin\theta)^{\bar{\lambda}}$$
$$+b_{1i}\mu_{2i}r^{\lambda}(\cos\theta+\mu_{2i}\sin\theta)^{\lambda}+b_{2i}\mu_{2i}r^{\bar{\lambda}}(\cos\theta+\mu_{2i}\sin\theta)^{\bar{\lambda}}\Big\}$$
$$=2\,\mathrm{Re}\Big\{a_{1i}\mu_{1i}r^{\lambda}(\cos\theta+\mu_{1i}\sin\theta)^{\lambda}+\bar{a}_{2i}\bar{\mu}_{1i}r^{\lambda}(\cos\theta+\bar{\mu}_{1i}\sin\theta)^{\lambda}$$
$$+b_{1i}\mu_{2i}r^{\lambda}(\cos\theta+\mu_{2i}\sin\theta)^{\lambda}+\bar{b}_{2i}\bar{\mu}_{2i}r^{\lambda}(\cos\theta+\bar{\mu}_{2i}\sin\theta)^{\lambda}\Big\} \tag{37}$$

2) $$P_{yi}=-2\,\mathrm{Re}\left\{\phi_{ai}(z_{1i})+\phi_{bi}(z_{2i})\right\}$$
$$=-2\,\mathrm{Re}\Big\{a_{1i}r^{\lambda}(\cos\theta+\mu_{1i}\sin\theta)^{\lambda}+a_{2i}r^{\bar{\lambda}}(\cos\theta+\mu_{1i}\sin\theta)^{\bar{\lambda}}$$
$$+b_{1i}r^{\lambda}(\cos\theta+\mu_{2i}\sin\theta)^{\lambda}+b_{2i}r^{\bar{\lambda}}(\cos\theta+\mu_{2i}\sin\theta)^{\bar{\lambda}}\Big\}$$
$$=-2\,\mathrm{Re}\Big\{a_{1i}r^{\lambda}(\cos\theta+\mu_{1i}\sin\theta)^{\lambda}+\bar{a}_{2i}r^{\lambda}(\cos\theta+\bar{\mu}_{1i}\sin\theta)^{\lambda}$$
$$+b_{1i}r^{\lambda}(\cos\theta+\mu_{2i}\sin\theta)^{\lambda}+\bar{b}_{2i}r^{\lambda}(\cos\theta+\bar{\mu}_{2i}\sin\theta)^{\lambda}\Big\} \tag{38}$$

3) $$u_{xi}=2\,\mathrm{Re}\left\{p_{1i}\phi_{ai}(z_{1i})+p_{2i}\phi_{bi}(z_{2i})\right\}$$
$$=2\,\mathrm{Re}\Big\{a_{1i}p_{1i}r^{\lambda}(\cos\theta+\mu_{1i}\sin\theta)^{\lambda}+a_{2i}p_{1i}r^{\bar{\lambda}}(\cos\theta+\mu_{1i}\sin\theta)^{\bar{\lambda}}$$
$$+b_{1i}p_{2i}r^{\lambda}(\cos\theta+\mu_{2i}\sin\theta)^{\lambda}+b_{2i}p_{2i}r^{\bar{\lambda}}(\cos\theta+\mu_{2i}\sin\theta)^{\bar{\lambda}}\Big\}$$
$$=2\,\mathrm{Re}\Big\{a_{1i}p_{1i}r^{\lambda}(\cos\theta+\mu_{1i}\sin\theta)^{\lambda}+\bar{a}_{2i}\bar{p}_{1i}r^{\lambda}(\cos\theta+\bar{\mu}_{1i}\sin\theta)^{\lambda}$$
$$+b_{1i}p_{2i}r^{\lambda}(\cos\theta+\mu_{2i}\sin\theta)^{\lambda}+\bar{b}_{2i}\bar{p}_{2i}r^{\lambda}(\cos\theta+\bar{\mu}_{2i}\sin\theta)^{\lambda}\Big\} \tag{39}$$

4) $$u_{yi}=2\,\mathrm{Re}\left\{q_{1i}\phi_{ai}(z_{1i})+q_{2i}\phi_{bi}(z_{2i})\right\}$$
$$=2\,\mathrm{Re}\Big\{a_{1i}q_{1i}r^{\lambda}(\cos\theta+\mu_{1i}\sin\theta)^{\lambda}+a_{2i}q_{1i}r^{\bar{\lambda}}(\cos\theta+\mu_{1i}\sin\theta)^{\bar{\lambda}}$$
$$+b_{1i}q_{2i}r^{\lambda}(\cos\theta+\mu_{2i}\sin\theta)^{\lambda}+b_{2i}q_{2i}r^{\bar{\lambda}}(\cos\theta+\mu_{2i}\sin\theta)^{\bar{\lambda}}\Big\}$$
$$=2\,\mathrm{Re}\Big\{a_{1i}q_{1i}r^{\lambda}(\cos\theta+\mu_{1i}\sin\theta)^{\lambda}+\bar{a}_{2i}\bar{q}_{1i}r^{\lambda}(\cos\theta+\bar{\mu}_{1i}\sin\theta)^{\lambda}$$
$$+b_{1i}q_{2i}r^{\lambda}(\cos\theta+\mu_{2i}\sin\theta)^{\lambda}+\bar{b}_{2i}\bar{q}_{2i}r^{\lambda}(\cos\theta+\bar{\mu}_{2i}\sin\theta)^{\lambda}\Big\} \tag{40}$$

5) $$\sigma_{xi}=2\,\mathrm{Re}\left\{\mu_{1i}^{2}\phi_{ai}'(z_{1i})+\mu_{2i}^{2}\phi_{bi}'(z_{2i})\right\}$$
$$=2\,\mathrm{Re}\Big\{a_{1i}\mu_{1i}^{2}\lambda r^{\lambda-1}(\cos\theta+\mu_{1i}\sin\theta)^{\lambda-1}+a_{2i}\mu_{1i}^{2}\bar{\lambda}r^{\bar{\lambda}-1}(\cos\theta+\mu_{1i}\sin\theta)^{\bar{\lambda}-1}$$
$$+b_{1i}\mu_{2i}^{2}\lambda r^{\lambda-1}(\cos\theta+\mu_{2i}\sin\theta)^{\lambda-1}+b_{2i}\mu_{2i}^{2}\bar{\lambda}r^{\bar{\lambda}-1}(\cos\theta+\mu_{2i}\sin\theta)^{\bar{\lambda}-1}\Big\}$$
$$=2\,\mathrm{Re}\Big\{a_{1i}\mu_{1i}^{2}\lambda r^{\lambda-1}(\cos\theta+\mu_{1i}\sin\theta)^{\lambda-1}+\bar{a}_{2i}\bar{\mu}_{1i}^{2}\lambda r^{\lambda-1}(\cos\theta+\bar{\mu}_{1i}\sin\theta)^{\lambda-1}$$
$$+b_{1i}\mu_{2i}^{2}\lambda r^{\lambda-1}(\cos\theta+\mu_{2i}\sin\theta)^{\lambda-1}+\bar{b}_{2i}\bar{\mu}_{2i}^{2}\lambda r^{\lambda-1}(\cos\theta+\bar{\mu}_{2i}\sin\theta)^{\lambda-1}\Big\} \tag{41}$$

6) $$\sigma_{yi}=2\,\mathrm{Re}\left\{\phi_{ai}'(z_{1i})+\phi_{bi}'(z_{2i})\right\}$$
$$=2\,\mathrm{Re}\Big\{a_{1i}\lambda r^{\lambda-1}(\cos\theta+\mu_{1i}\sin\theta)^{\lambda-1}+a_{2i}\bar{\lambda}r^{\bar{\lambda}-1}(\cos\theta+\mu_{1i}\sin\theta)^{\bar{\lambda}-1}$$
$$+b_{1i}\lambda r^{\lambda-1}(\cos\theta+\mu_{2i}\sin\theta)^{\lambda-1}+b_{2i}\bar{\lambda}r^{\bar{\lambda}-1}(\cos\theta+\mu_{2i}\sin\theta)^{\bar{\lambda}-1}\Big\}$$
$$=2\,\mathrm{Re}\Big\{a_{1i}\lambda r^{\lambda-1}(\cos\theta+\mu_{1i}\sin\theta)^{\lambda-1}+\bar{a}_{2i}\lambda r^{\lambda-1}(\cos\theta+\bar{\mu}_{1i}\sin\theta)^{\lambda-1}$$
$$+b_{1i}\lambda r^{\lambda-1}(\cos\theta+\mu_{2i}\sin\theta)^{\lambda-1}+\bar{b}_{2i}\lambda r^{\lambda-1}(\cos\theta+\bar{\mu}_{2i}\sin\theta)^{\lambda-1}\Big\} \tag{42}$$

7) $$\tau_{xyi}=-2\,\mathrm{Re}\left\{\mu_{1i}\phi_{ai}'(z_{1i})+\mu_{2i}\phi_{bi}'(z_{2i})\right\}$$
$$=-2\,\mathrm{Re}\Big\{a_{1i}\mu_{1i}\lambda r^{\lambda-1}(\cos\theta+\mu_{1i}\sin\theta)^{\lambda-1}+a_{2i}\mu_{1i}\bar{\lambda}r^{\bar{\lambda}-1}(\cos\theta+\mu_{1i}\sin\theta)^{\bar{\lambda}-1}$$
$$+b_{1i}\mu_{2i}\lambda r^{\lambda-1}(\cos\theta+\mu_{2i}\sin\theta)^{\lambda-1}+b_{2i}\mu_{2i}\bar{\lambda}r^{\bar{\lambda}-1}(\cos\theta+\mu_{2i}\sin\theta)^{\bar{\lambda}-1}\Big\}$$
$$=-2\,\mathrm{Re}\Big\{a_{1i}\mu_{1i}\lambda r^{\lambda-1}(\cos\theta+\mu_{1i}\sin\theta)^{\lambda-1}+\bar{a}_{2i}\bar{\mu}_{1i}\lambda r^{\lambda-1}(\cos\theta+\bar{\mu}_{1i}\sin\theta)^{\lambda-1}$$
$$+b_{1i}\mu_{2i}\lambda r^{\lambda-1}(\cos\theta+\mu_{2i}\sin\theta)^{\lambda-1}+\bar{b}_{2i}\bar{\mu}_{2i}\lambda r^{\lambda-1}(\cos\theta+\bar{\mu}_{2i}\sin\theta)^{\lambda-1}\Big\} \tag{43}$$

以上をまとめると次のようになる.

$$\left.\begin{aligned}
u_{xi} &= 2\,\mathrm{Re}\left[r^{\lambda}\left\{a_{1i}p_{1i}d_{i1}^{\lambda} + \overline{a}_{2i}\overline{p}_{1i}\overline{d}_{i1}^{\lambda} + b_{1i}p_{2i}d_{i2}^{\lambda} + \overline{b}_{2i}\overline{p}_{2i}\overline{d}_{i2}^{\lambda}\right\}\right] \\
u_{yi} &= 2\,\mathrm{Re}\left[r^{\lambda}\left\{a_{1i}q_{1i}d_{i1}^{\lambda} + \overline{a}_{2i}\overline{q}_{1i}\overline{d}_{i1}^{\lambda} + b_{1i}q_{2i}d_{i2}^{\lambda} + \overline{b}_{2i}\overline{q}_{2i}\overline{d}_{i2}^{\lambda}\right\}\right] \\
\sigma_{xi} &= 2\,\mathrm{Re}\left[\lambda r^{\lambda-1}\left\{a_{1i}\mu_{1i}^{2}d_{i1}^{\lambda-1} + \overline{a}_{2i}\overline{\mu}_{1i}^{2}\overline{d}_{i1}^{\lambda-1} + b_{1i}\mu_{2i}^{2}d_{i2}^{\lambda-1} + \overline{b}_{2i}\overline{\mu}_{2i}^{2}\overline{d}_{i2}^{\lambda-1}\right\}\right] \\
\sigma_{yi} &= 2\,\mathrm{Re}\left[\lambda r^{\lambda-1}\left\{a_{1i}d_{i1}^{\lambda-1} + \overline{a}_{2i}\overline{d}_{i1}^{\lambda-1} + b_{1i}d_{i2}^{\lambda-1} + \overline{b}_{2i}\overline{d}_{i2}^{\lambda-1}\right\}\right] \\
\tau_{xyi} &= -2\,\mathrm{Re}\left[\lambda r^{\lambda-1}\left\{a_{1i}\mu_{1i}d_{i1}^{\lambda-1} + \overline{a}_{2i}\overline{\mu}_{1i}\overline{d}_{i1}^{\lambda-1} + b_{1i}\mu_{2i}d_{i2}^{\lambda-1} + \overline{b}_{2i}\overline{\mu}_{2i}\overline{d}_{i2}^{\lambda-1}\right\}\right]
\end{aligned}\right\} \tag{44}$$

ここに

$$\left.\begin{aligned}
d_{i1} &= \cos\theta_i + \mu_{1i}\sin\theta_2, & \overline{d}_{i1} &= \cos\theta_i + \overline{\mu}_{1i}\sin\theta_i, \\
d_{i2} &= \cos\theta_i + \mu_{2i}\sin\theta_{2i}, & \overline{d}_{i2} &= \cos\theta_i + \overline{\mu}_{2i}\sin\theta_2, \\
d_{i3} &= \cos\theta_{i+1} + \mu_{1i}\sin\theta_{i+1}, & \overline{d}_{i3} &= \cos\theta_{i+1} + \overline{\mu}_{1i}\sin\theta_{i+1}, \\
d_{i4} &= \cos\theta_{i+1} + \mu_{2i}\sin\theta_{i+1}, & \overline{d}_{i4} &= \cos\theta_{i+1} + \overline{\mu}_{2i}\sin\theta_{i+1}.
\end{aligned}\right\} \tag{45}$$

これらを極座標表示すれば次のようになる.

$$\left.\begin{aligned}
&1)\quad P_{ri} = P_{xi}\cdot e^{-i\theta} \\
&2)\quad P_{\theta i} = P_{yi}\cdot e^{-i\theta} \\
&3)\quad u_{ri} = u_{xi}\cdot e^{-i\theta} \\
&4)\quad u_{\theta i} = u_{yi}\cdot e^{-i\theta} \\
&5)\quad \sigma_{ri} = \sigma_{xi}\cdot e^{2i\theta} \\
&6)\quad \sigma_{\theta i} = \sigma_{yi}\cdot e^{2i\theta} \\
&7)\quad \tau_{r\theta i} = \tau_{xyi}\cdot e^{2i\theta}
\end{aligned}\right\} \tag{46}$$

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